Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/18-1.1.1.7-P-x-a+b-x-^m-c+d-x-^n-e+f-x-^p-g+h-x-^q

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [35]. This is test number [18].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (35)	0.00 (0)
Mathematica	100.00 (35)	0.00 (0)
Maple	100.00 (35)	0.00 (0)
Fricas	25.71 (9)	74.29 (26)
Mupad	0.00 (0)	100.00 (35)
Giac	0.00 (0)	100.00 (35)
Maxima	0.00 (0)	100.00 (35)
Sympy	0.00 (0)	100.00 (35)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

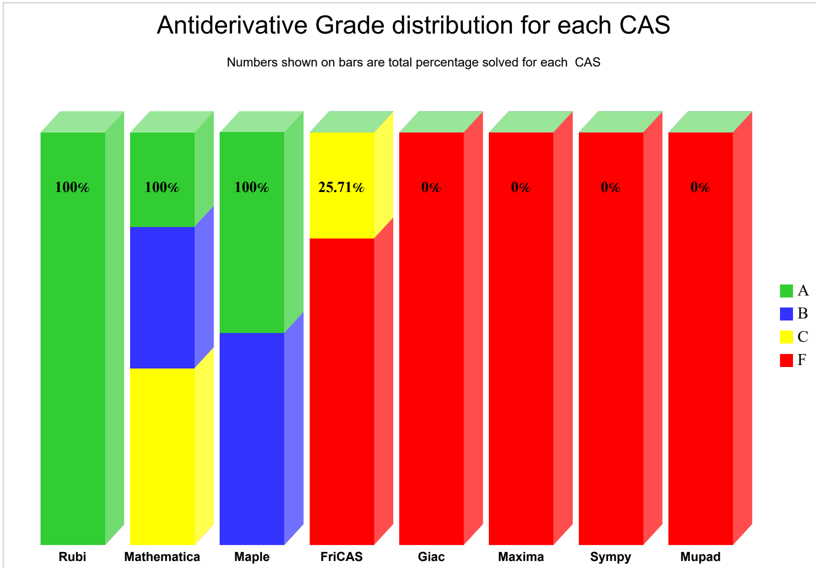
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

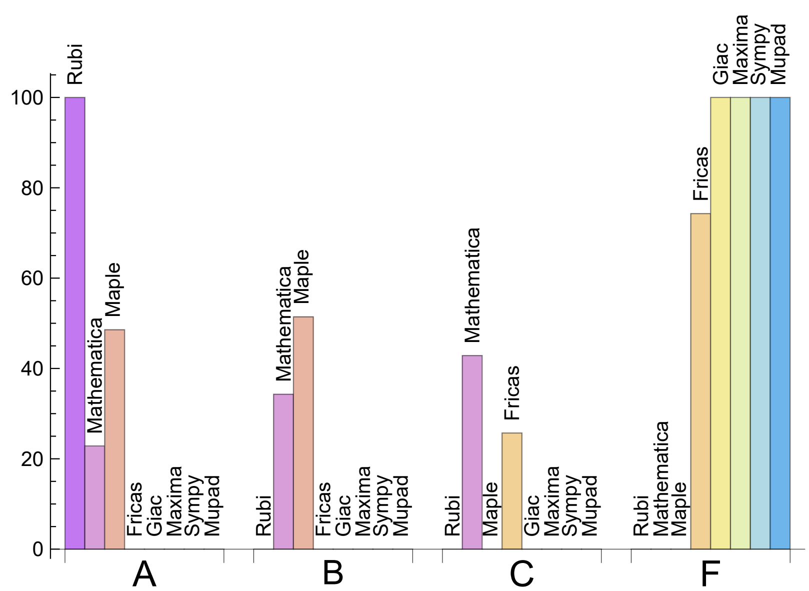
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	48.571	51.429	0.000	0.000
Mathematica	22.857	34.286	42.857	0.000
Fricas	0.000	0.000	25.714	74.286
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	0.000	0.000	100.000
Maxima	0.000	0.000	0.000	100.000
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	26	26.92	73.08	0.00
Mupad	35	0.00	100.00	0.00
Giac	35	97.14	0.00	2.86
Maxima	35	100.00	0.00	0.00
Sympy	35	74.29	25.71	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.14
Rubi	1.86
Maple	5.25
Mathematica	31.13
Sympy	-nan(ind)
Maxima	-nan(ind)
Giac	-nan(ind)
Mupad	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	697.77	1.01	686.00	1.00
Fricas	1007.22	1.98	859.00	2.08
Maple	1504.14	2.10	1238.00	1.85
Mathematica	5806.66	6.15	825.00	1.34
Sympy	-nan(ind)	-nan(ind)	nan	nan
Maxima	-nan(ind)	-nan(ind)	nan	nan
Giac	-nan(ind)	-nan(ind)	nan	nan
Mupad	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

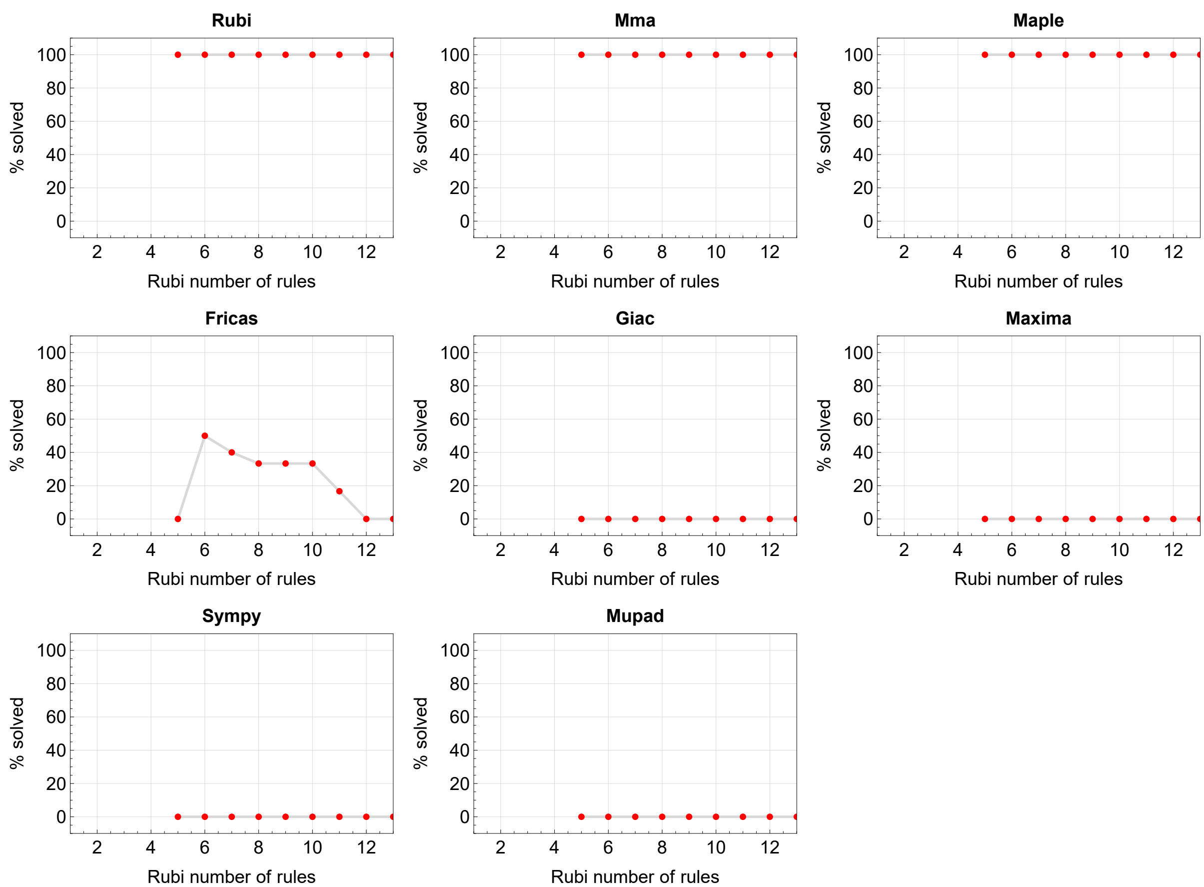


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

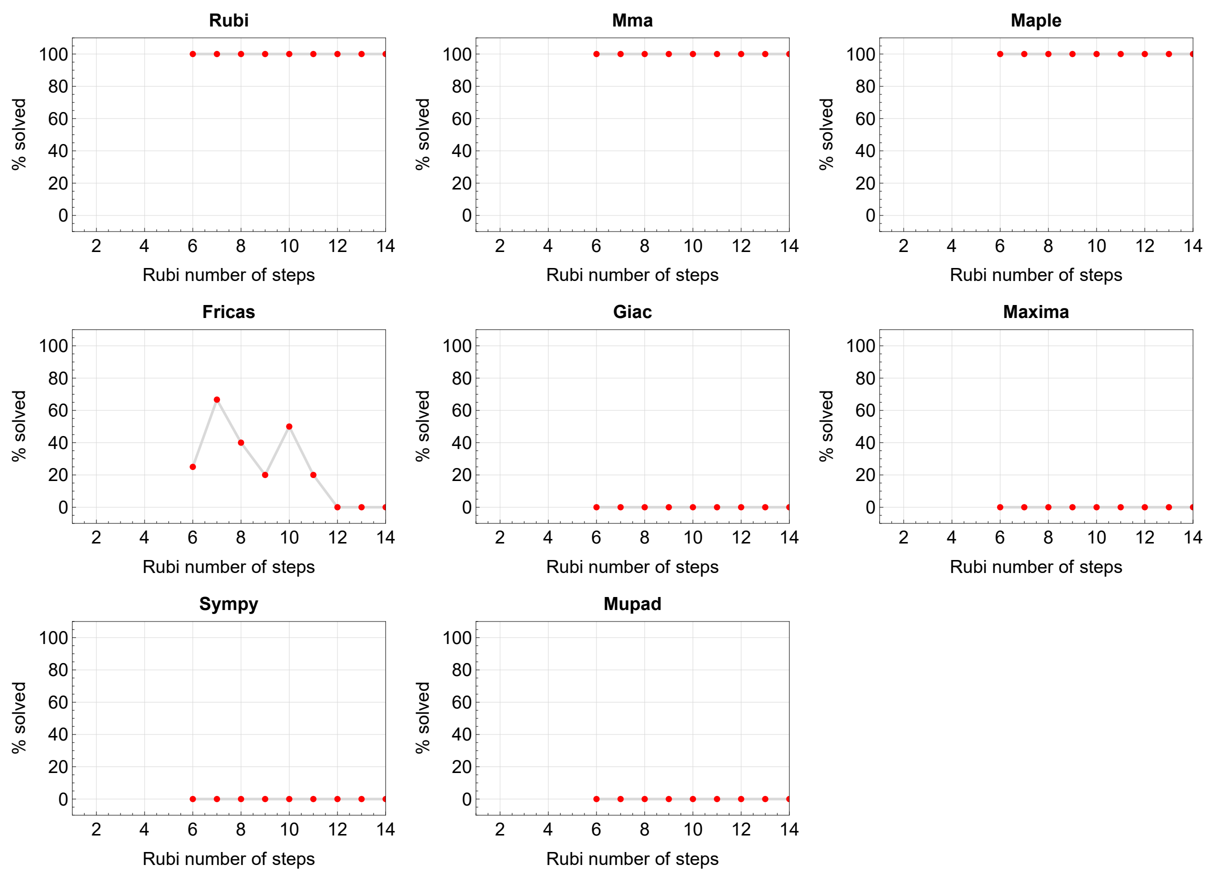


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

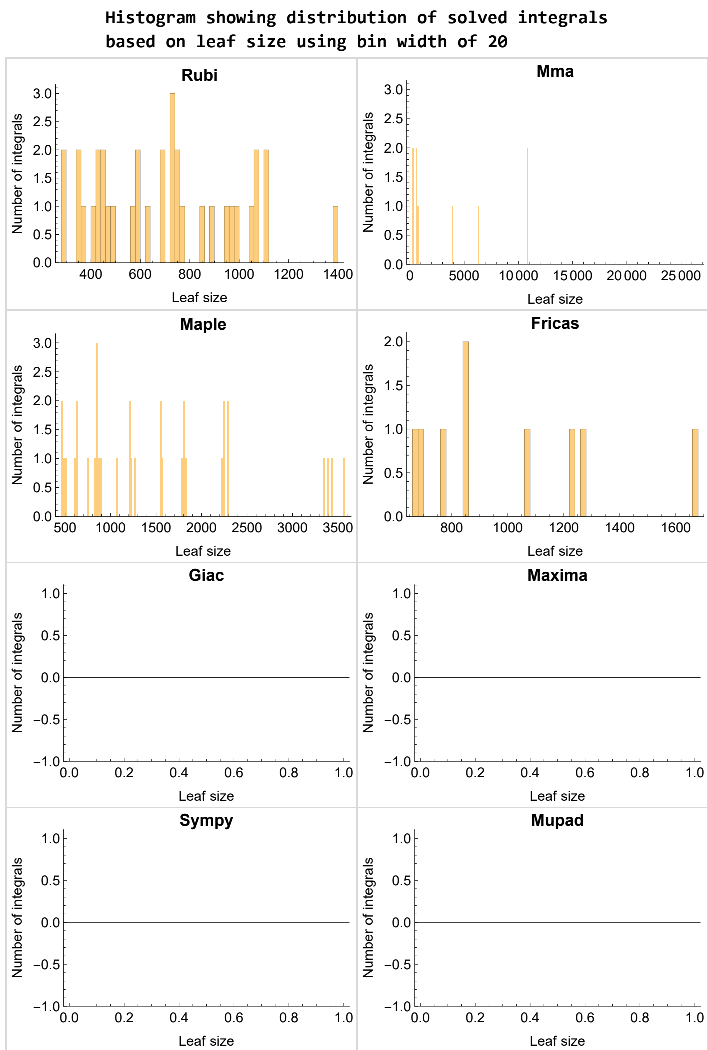


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

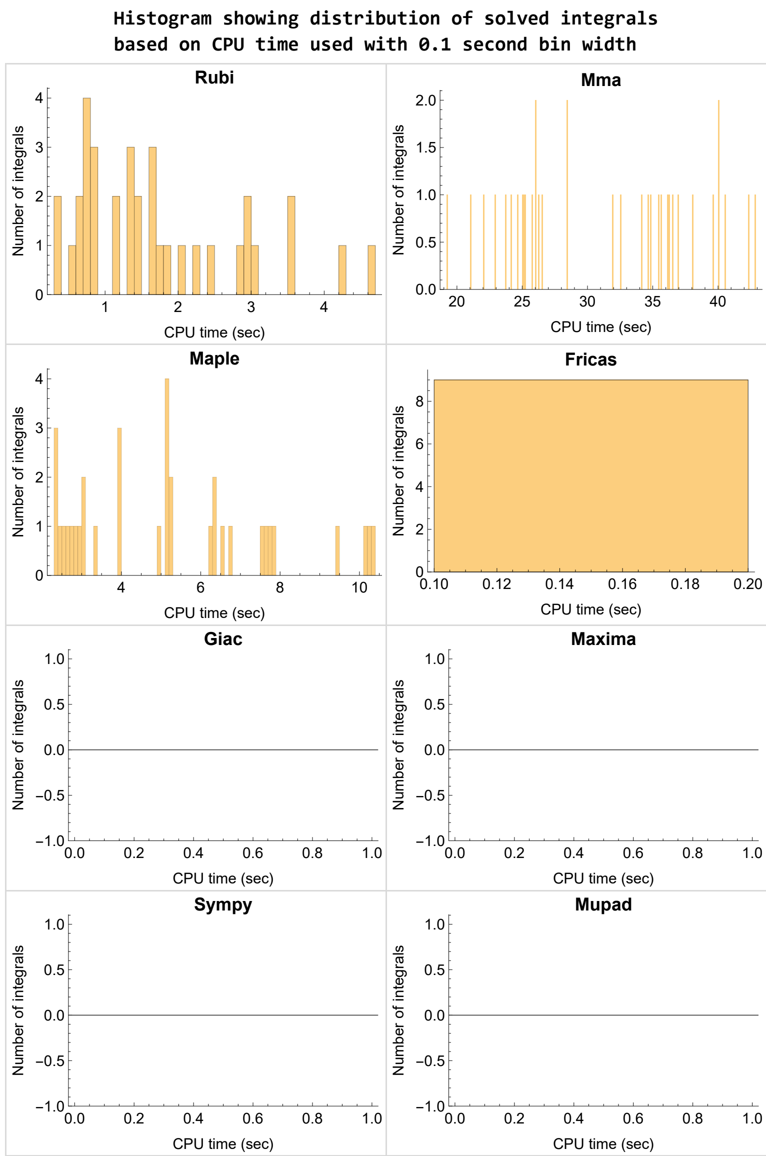


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

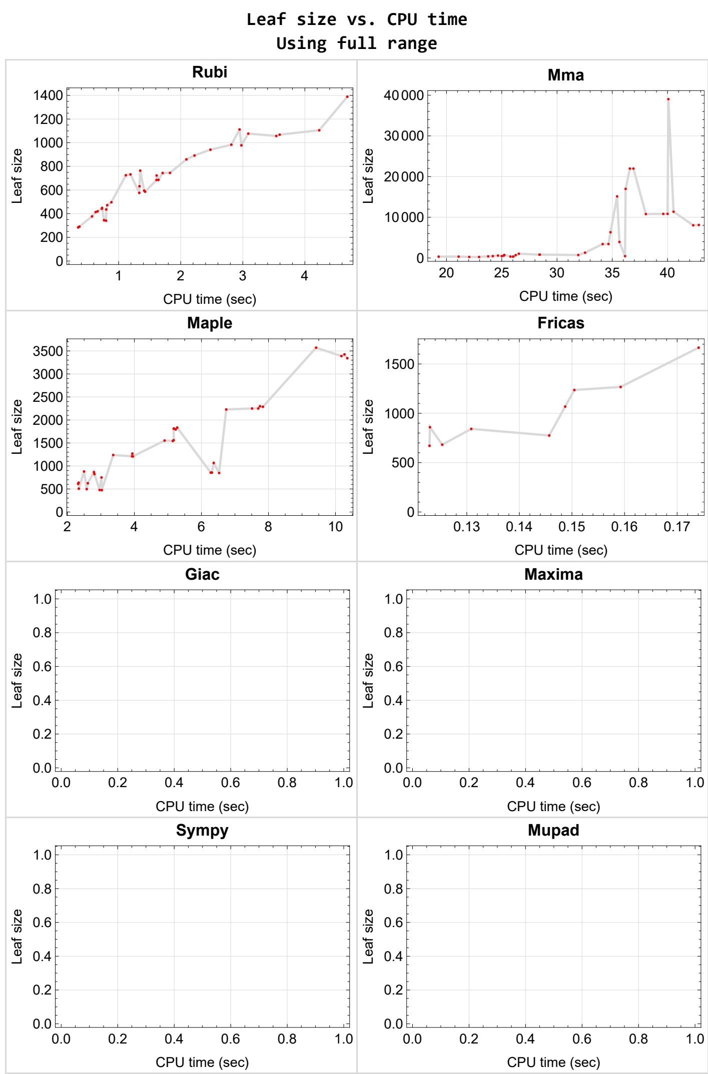


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{6, 11, 21, 25, 31, 32, 34, 35\}$

Mathematica $\{6, 7, 11, 21, 22, 31, 32, 34\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

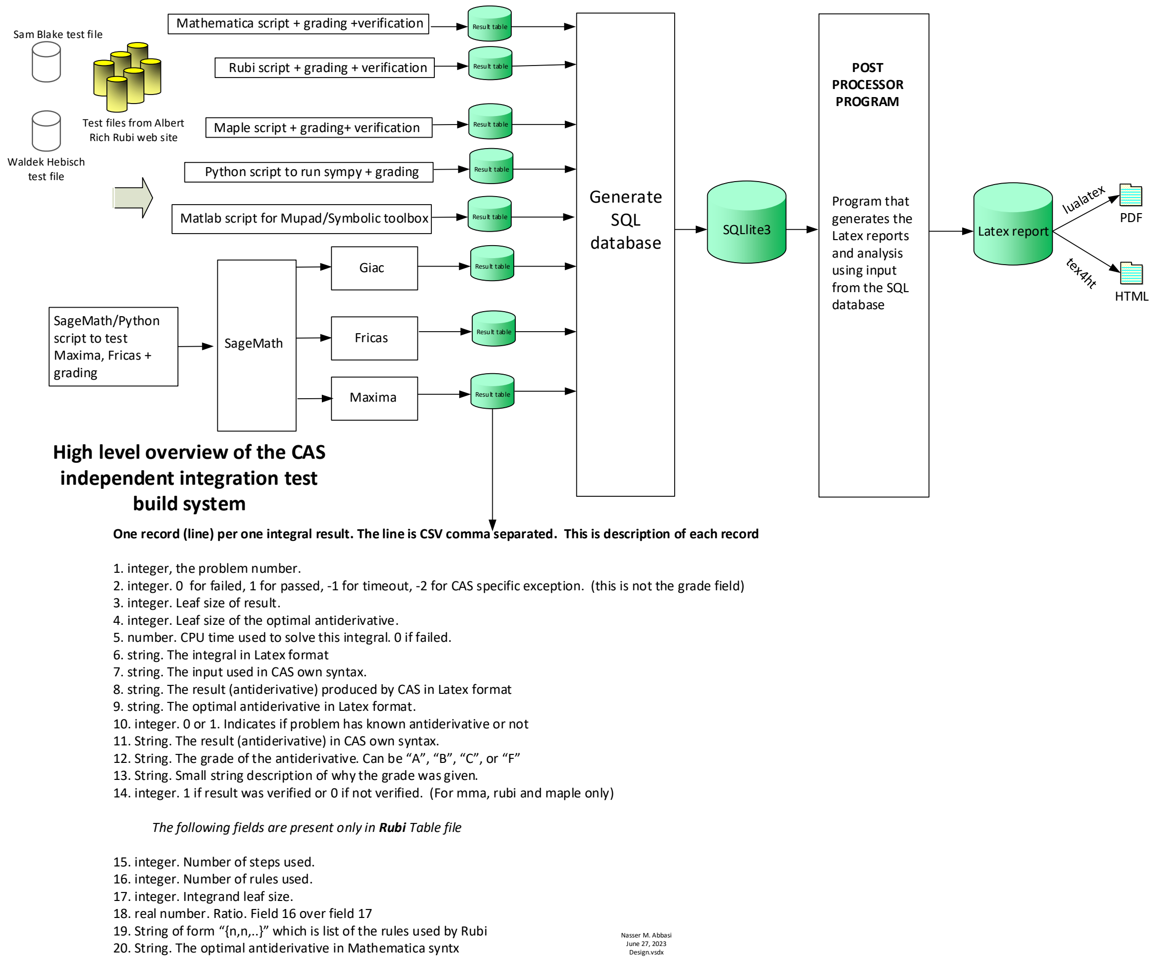
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 8, 9, 12, 13, 14, 23, 24, 34 }

B grade { 6, 7, 10, 11, 15, 21, 22, 25, 31, 32, 33, 35 }

C grade { 1, 2, 3, 4, 5, 16, 17, 18, 19, 20, 26, 27, 28, 29, 30 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 16, 17, 18, 19, 20, 26, 27, 28, 29, 30, 31, 33 }

B grade { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 21, 22, 23, 24, 25, 32, 34, 35 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { }

B grade { }

C grade { 1, 2, 3, 16, 17, 18, 26, 27, 28 }

F normal fail { 9, 10, 14, 15, 24, 25, 35 }

F(-1) timedout fail { 4, 5, 6, 7, 8, 11, 12, 13, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35 }

F(-1) timedout fail { }

F(-2) exception fail { 34 }

2.1.7 Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 26, 27, 28, 29, 31, 32, 33, 34 }

F(-1) timedout fail { 5, 10, 15, 19, 20, 24, 25, 30, 35 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	700	723	806	866	0	1236	0	0	0
N.S.	1	1.03	1.15	1.24	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	1.705	28.405	2.793	0.000	0.150	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	414	450	625	0	842	0	0	0
N.S.	1	1.02	1.11	1.54	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.656	24.188	2.610	0.000	0.131	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	319	498	0	671	0	0	0
N.S.	1	1.00	1.12	1.75	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.359	19.292	2.578	0.000	0.123	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	345	245	478	0	0	0	0	0
N.S.	1	1.10	0.78	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.808	22.947	2.962	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	678	685	3412	1208	0	0	0	0	0
N.S.	1	1.01	5.03	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.697	34.128	3.956	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	981	983	21961	1814	0	0	0	0	0
N.S.	1	1.00	22.39	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.958	36.590	5.178	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	736	725	8030	1544	0	0	0	0	0
N.S.	1	0.99	10.91	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.208	42.322	5.146	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	586	848	0	0	0	0	0
N.S.	1	1.00	1.33	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.728	24.657	6.528	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	606	576	333	2250	0	0	0	0	0
N.S.	1	0.95	0.55	3.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.366	26.023	7.694	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1081	1068	10828	3389	0	0	0	0	0
N.S.	1	0.99	10.02	3.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.684	39.612	10.176	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	898	892	15131	1809	0	0	0	0	0
N.S.	1	0.99	16.85	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.378	35.437	5.174	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	472	472	443	1560	0	0	0	0	0
N.S.	1	1.00	0.94	3.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.844	36.156	5.172	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	723	855	0	0	0	0	0
N.S.	1	1.00	1.61	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.788	25.235	6.278	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	595	341	2298	0	0	0	0	0
N.S.	1	0.95	0.55	3.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.488	25.784	7.746	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1090	1077	10790	3571	0	0	0	0	0
N.S.	1	0.99	9.90	3.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.300	38.038	9.421	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	721	744	825	880	0	1267	0	0	0
N.S.	1	1.03	1.14	1.22	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	1.833	28.445	2.498	0.000	0.159	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	420	442	637	0	859	0	0	0
N.S.	1	1.02	1.08	1.55	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.746	25.003	2.336	0.000	0.123	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	326	506	0	682	0	0	0
N.S.	1	1.00	1.12	1.74	0.00	2.34	0.00	0.00	0.00
time (sec)	N/A	0.403	21.099	2.342	0.000	0.125	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	341	249	475	0	0	0	0	0
N.S.	1	1.10	0.81	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.852	22.056	3.028	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	686	3419	1211	0	0	0	0	0
N.S.	1	1.01	5.03	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.762	34.657	3.928	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	980	978	21961	1834	0	0	0	0	0
N.S.	1	1.00	22.41	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.125	36.913	5.277	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	734	732	8107	1552	0	0	0	0	0
N.S.	1	1.00	11.04	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.257	42.832	4.901	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	583	856	0	0	0	0	0
N.S.	1	1.00	1.34	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.835	25.181	6.313	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	616	586	340	2249	0	0	0	0	0
N.S.	1	0.95	0.55	3.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.493	26.043	7.511	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1128	1105	10836	3425	0	0	0	0	0
N.S.	1	0.98	9.61	3.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.573	40.007	10.267	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1097	1112	1291	1238	0	1665	0	0	0
N.S.	1	1.01	1.18	1.13	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	3.288	32.538	3.368	0.000	0.174	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	632	686	824	0	1068	0	0	0
N.S.	1	1.03	1.12	1.35	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	1.511	26.261	2.808	0.000	0.149	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	377	390	611	0	775	0	0	0
N.S.	1	1.02	1.06	1.66	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.680	23.770	2.322	0.000	0.146	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	465	497	1036	750	0	0	0	0	0
N.S.	1	1.07	2.23	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.998	26.549	3.018	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	738	745	3935	1269	0	0	0	0	0
N.S.	1	1.01	5.33	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.078	35.647	3.939	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1395	1389	39032	2228	0	0	0	0	0
N.S.	1	1.00	27.98	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.284	40.077	6.740	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	937	941	16972	1794	0	0	0	0	0
N.S.	1	1.00	18.11	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.686	36.207	5.232	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	757	764	6321	1065	0	0	0	0	0
N.S.	1	1.01	8.35	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.547	34.840	6.366	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	867	859	721	2286	0	0	0	0	0
N.S.	1	0.99	0.83	2.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.348	31.921	7.834	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1070	1057	11363	3342	0	0	0	0	0
N.S.	1	0.99	10.62	3.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.799	40.542	10.352	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [5] had the largest ratio of [.3250000000000000011]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	9	1.03	40	0.225
2	A	7	7	1.02	38	0.184
3	A	6	6	1.00	33	0.182
4	A	10	9	1.10	40	0.225
5	A	14	13	1.01	40	0.325
6	A	12	11	1.00	42	0.262
7	A	8	7	0.99	42	0.167
8	A	6	5	1.00	42	0.119
9	A	8	7	0.95	42	0.167
10	A	11	10	0.99	42	0.238
11	A	12	11	0.99	49	0.224
12	A	6	5	1.00	49	0.102
13	A	6	5	1.00	49	0.102
14	A	8	7	0.95	49	0.143
15	A	9	8	0.99	49	0.163
16	A	10	10	1.03	58	0.172
17	A	8	8	1.02	53	0.151
18	A	7	7	1.00	60	0.117
19	A	11	10	1.10	60	0.167
20	A	14	13	1.01	60	0.217
21	A	13	12	1.00	62	0.194
22	A	9	8	1.00	62	0.129

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	1.00	62	0.097
24	A	9	8	0.95	62	0.129
25	A	10	9	0.98	62	0.145
26	A	11	11	1.01	42	0.262
27	A	10	10	1.03	40	0.250
28	A	8	8	1.02	35	0.229
29	A	12	11	1.07	42	0.262
30	A	14	13	1.01	42	0.310
31	A	12	11	1.00	44	0.250
32	A	11	10	1.00	44	0.227
33	A	9	8	1.01	44	0.182
34	A	12	11	0.99	44	0.250
35	A	11	10	0.99	44	0.227

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.1.1 Optimal result

Integrand size = 40, antiderivative size = 700

$$\begin{aligned}
&\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx \\
&= \frac{2b(7aBdfh+b(5Adfh-4B(dfg+deh+cfh)))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2} \\
&\quad + \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} \\
&\quad + \frac{2\sqrt{-de+cf}(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(dfg+deh+cfh))-b^2(10Adfh(dfg+deh+cfh)}{15d^3f^{5/2}} \\
&\quad - \frac{2\sqrt{-de+cf}(15a^2d^2f^2h^2(Bg-Ah)+10abdfh(3Adfgh-B(ch(fg-eh)+dg(2fg+eh)))-b^2(5Adf}{}
\end{aligned}$$

output
```
2/15*b*(7*a*B*d*f*h+b*(5*A*d*f*h-4*B*(c*f*h+d*e*h+d*f*g)))*(d*x+c)^(1/2)*(
f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+2/5*b*B*(b*x+a)*(d*x+c)^(1/2)*(f*x+
e)^(1/2)*(h*x+g)^(1/2)/d/f/h+2/15*(15*a^2*B*d^2*f^2*h^2+10*a*b*d*f*h*(3*A*
d*f*h-2*B*(c*f*h+d*e*h+d*f*g))-b^2*(10*A*d*f*h*(c*f*h+d*e*h+d*f*g)-B*(8*c^
2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f^2*g^2))))*Ellip
ticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/
2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^3/f^(5/2)
/h^3/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2/15*(15*a^2*d^2*f^2*h^2*(
-A*h+B*g)+10*a*b*d*f*h*(3*A*d*f*g*h-B*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g)))-b^
2*(5*A*d*f*h*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g))-B*(4*c^2*f*h^2*(-e*h+f*g)+c*
d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2*h^2+3*e*f*g*h+8*f^2*g^2)))
)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*
g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d
*g))^(1/2)/d^3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.1.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.41 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(-d^2\sqrt{-c+\frac{de}{f}}(15a^2Bd^2f^2h^2-10abdfh(-3Adfh+2B(dfg+deh+cfh))+b^2(-10Adfh(dfg+de\right.}{}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
(-2*(-(d^2*Sqrt[-c + (d*e)/f]*(15*a^2*B*d^2*f^2*h^2 - 10*a*b*d*f*h*(-3*A*d
*f*h + 2*B*(d*f*g + d*e*h + c*f*h)) + b^2*(-10*A*d*f*h*(d*f*g + d*e*h + c*
f*h) + B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g
*h + 8*e^2*h^2))))*(e + f*x)*(g + h*x)) + b*d^2*Sqrt[-c + (d*e)/f]*f*h*(c
+ d*x)*(e + f*x)*(g + h*x)*(-5*A*b*d*f*h - 10*a*B*d*f*h + b*B*(4*c*f*h + d
*(4*f*g + 4*e*h - 3*f*h*x))) - I*(d*e - c*f)*h*(15*a^2*B*d^2*f^2*h^2 - 10*
a*b*d*f*h*(-3*A*d*f*h + 2*B*(d*f*g + d*e*h + c*f*h)) + b^2*(-10*A*d*f*h*(d
*f*g + d*e*h + c*f*h) + B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*
f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*
(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c
+ (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*d*h*(15*a^
2*d^2*f^2*(-(B*e) + A*f)*h^2 + 10*a*b*d*f*h*(-3*A*d*e*f*h + B*c*f*(-(f*g)
+ e*h) + B*d*e*(f*g + 2*e*h)) - b^2*(-5*A*d*f*h*(c*f*(-(f*g) + e*h) + d*e*
(f*g + 2*e*h)) + B*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*(-4*f^2*g^2 + e*f*g
*h + 3*e^2*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(
3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*E
llipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e
*h - c*f*h)]))/(15*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqrt[c + d*x]*Sqrt[e + f
*x]*Sqrt[g + h*x])
```

3.1.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2100, 2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \; 2100$$

$$\frac{\int \frac{5Adfha^2+b(5Abdfh+7aBdfh-4bB(dfg+deh+cfh))x^2-bB(2bceg+a(deg+cfg+ceh))+(5Bdfha^2+2b(5Adfh-B(dfg+deh+cfh))a-3b^2B(deg}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{5dfh}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \; 2118$$

$$2\int \frac{d\left(-\left(\left(5Adfh(deg+cfg+ceh)-B\left(4fh(fg+eh)c^2+2d\left(2f^2g^2+3efhg+2e^2h^2\right)c+4d^2eg(fg+eh)\right)\right)b^2\right)-10aBdfh(deg+cfg+ceh)b+15a^2Ad^2f^2h^2+\left(-\left(\left(10Adfh}{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+h}}$$

$$3d^2fh$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 27

$$\int \frac{-\left(\left(5Adfh(deg+cfg+ceh)-B\left(4fh(fg+eh)c^2+2d\left(2f^2g^2+3efhg+2e^2h^2\right)c+4d^2eg(fg+eh)\right)\right)b^2\right)-10aBdfh(deg+cfg+ceh)b+15a^2Ad^2f^2h^2+\left(-\left(\left(10Adfh(dfg}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}$$

$$3dfh$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 176

$$\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h^2+7cdfh(eh+fg)+d^2\left(8e^2h^2+7efgh+8f^2g^2\right)\right)\right)\right)\right)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+}}$$

$$h$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 124

$$\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h^2+7cdfh(eh+fg)+d^2\left(8e^2h^2+7efgh+8f^2g^2\right)\right)\right.\right.\right.$$

$$h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 123

$$\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h^2\right.\right.\right.\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 131

3.1. $\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h^2\right.\right.\right.\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 131

$$\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h^2\right.\right.\right.\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 130

$$\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h^2\right.\right.\right.\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

input
```
Int[((a + b*x)^2*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

3.1. $\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*b*B*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) + ((
2*b*(5*A*b*d*f*h + 7*a*B*d*f*h - 4*b*B*(d*f*g + d*e*h + c*f*h))*Sqrt[c + d
*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((2*Sqrt[-(d*e) + c*f]*(15*a^
2*B*d^2*f^2*h^2 + 10*a*b*d*f*h*(3*A*d*f*h - 2*B*(d*f*g + d*e*h + c*f*h)) -
 b^2*(10*A*d*f*h*(d*f*g + d*e*h + c*f*h) - B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f
*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-
(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x
]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]*(15*a^2*d^2*f^2
*h^2*(B*g - A*h) + 10*a*b*d*f*h*(3*A*d*f*g*h - B*c*h*(f*g - e*h) - B*d*g*(
2*f*g + e*h)) - b^2*(5*A*d*f*h*(c*h*(f*g - e*h) + d*g*(2*f*g + e*h)) - B*(
4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 + e*f*g*h - 4*e^2*h^2) + d^2*g*
(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqr
t[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt
[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f
*x]*Sqrt[g + h*x]))/(3*d*f*h))/(5*d*f*h)
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

3.1. $\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.1.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2B\,b^2 x\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(b^2A+2abB-\frac{2B\,b^2(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{dfh}}{3}\right.}{}$
default	Expression too large to display

input
```
int((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2/5*B*b^2/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*
x+d*e*g*x+c*e*g)^(1/2)+2/3*(b^2*A+2*a*b*B-2/5*B*b^2/d/f/h*(2*c*f*h+2*d*e*h
+2*d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2*(a^2*A-2/5*B*b^2/d/f/h*c*e*g-2/3*(b^2*A+2*a*b*B-2/5
*B*b^2/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e
*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/
f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(
-g/h+c/d))^(1/2))+2*(2*a*b*A+a^2*B-2/5*B*b^2/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/
2*d*e*g)-2/3*(b^2*A+2*a*b*B-2/5*B*b^2/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f
/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h
+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f
*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)
/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g
/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

3.1.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1236, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="fricas")
```

output
```
2/45*(3*(3*B*b^2*d^3*f^3*h^3*x - 4*B*b^2*d^3*f^3*g*h^2 - (4*B*b^2*d^3*e*f^
2 + (4*B*b^2*c*d^2 - 5*(2*B*a*b + A*b^2)*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt
(f*x + e)*sqrt(h*x + g) - (8*B*b^2*d^3*f^3*g^3 + (3*B*b^2*d^3*e*f^2 + (3*B
*b^2*c*d^2 - 10*(2*B*a*b + A*b^2)*d^3)*f^3)*g^2*h + (3*B*b^2*d^3*e^2*f + (
3*B*b^2*c*d^2 - 5*(2*B*a*b + A*b^2)*d^3)*e*f^2 + (3*B*b^2*c^2*d - 5*(2*B*a
*b + A*b^2)*c*d^2 + 15*(B*a^2 + 2*A*a*b)*d^3)*f^3)*g*h^2 + (8*B*b^2*d^3*e^
3 + (3*B*b^2*c*d^2 - 10*(2*B*a*b + A*b^2)*d^3)*e^2*f + (3*B*b^2*c^2*d - 5*
(2*B*a*b + A*b^2)*c*d^2 + 15*(B*a^2 + 2*A*a*b)*d^3)*e*f^2 + (8*B*b^2*c^3 -
 45*A*a^2*d^3 - 10*(2*B*a*b + A*b^2)*c^2*d + 15*(B*a^2 + 2*A*a*b)*c*d^2)*f
^3)*h^3)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d
*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3
*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2
+ c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^
3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)) -
3*(8*B*b^2*d^3*f^3*g^2*h + (7*B*b^2*d^3*e*f^2 + (7*B*b^2*c*d^2 - 10*(2*B*a
*b + A*b^2)*d^3)*f^3)*g*h^2 + (8*B*b^2*d^3*e^2*f + (7*B*b^2*c*d^2 - 10*(2*
B*a*b + A*b^2)*d^3)*e*f^2 + (8*B*b^2*c^2*d - 10*(2*B*a*b + A*b^2)*c*d^2 +
15*(B*a^2 + 2*A*a*b)*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f
^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2
*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(...
```

3.1.6 Sympy [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\,(a+bx)^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**2*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),
x)
```

output
```
Integral((A + B*x)*(a + b*x)**2/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)
), x)
```

3.1.7 Maxima [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

3.1.8 Giac [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^2}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int(((A + B*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/
2)), x)
```

3.2 $\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.2.1 Optimal result

Integrand size = 38, antiderivative size = 405

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$
$$+\frac{2\sqrt{-de+cf}(3aBdfh+b(3Adfh-2B(dfg+deh+cfh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-}{f(dg}\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{2\sqrt{-de+cf}(3adfh(Bg-Ah)+b(3Adfgh-B(ch(fg-eh)+dg(2fg+eh))))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{Elli}}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

output

```
2/3*b*B*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+2/3*(3*a*B*d*f*h+b
*(3*A*d*f*h-2*B*(c*f*h+d*e*h+d*f*g)))*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f
-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/
(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(
-c*h+d*g))^(1/2)-2/3*(3*a*d*f*h*(-A*h+B*g)+b*(3*A*d*f*g*h-B*(c*h*(-e*h+f*g
)+d*g*(e*h+2*f*g))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*
f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)
*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.2.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.19 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\sqrt{c+dx}\left(2bBd^2fh(e+fx)(g+hx) - \frac{2d^2(-3Abdfh-3aBdfh+2bB(dfg+deh+cfh))(e+fx)(g+hx)}{c+dx} + \frac{2i(de-cf)h(3Abdfh+3\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((a + b*x)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(Sqrt[c + d*x]*(2*b*B*d^2*f*h*(e + f*x)*(g + h*x) - (2*d^2*(-3*A*b*d*f*h -
 3*a*B*d*f*h + 2*b*B*(d*f*g + d*e*h + c*f*h))*(e + f*x)*(g + h*x))/(c + d*
x) + ((2*I)*(d*e - c*f)*h*(3*A*b*d*f*h + 3*a*B*d*f*h - 2*b*B*(d*f*g + d*e*
h + c*f*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h
*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]],
 (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[-c + (d*e)/f] + ((2*I)*d*h*(3*a*d*
f*(-(B*e) + A*f)*h + b*(-3*A*d*e*f*h + B*c*f*(-(f*g) + e*h) + B*d*e*(f*g +
 2*e*h)))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x
))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (
d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[-c + (d*e)/f]))/(3*d^3*f^2*h^2*Sqrt[
e + f*x]*Sqrt[g + h*x])
```

3.2.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2097, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{ 2097}$$

$$\frac{\int \frac{3aAdfh-bB(deg+cfg+ceh)+(3Abdfh+3aBdfh-2bB(dfg+deh+cfh))x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3dfh}+\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\big\downarrow$ 176

$$\frac{\frac{(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}-\frac{(3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(eh+2fg)))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+h\cdots}}}{h}}{3dfh}$$

$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\big\downarrow$ 124

$$\frac{\frac{\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(eh+\cdots}{h}}{3dfh}$$

$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\big\downarrow$ 123

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-\cdots}{\cdots}}{3dfh}$$

$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\big\downarrow$ 131

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}(3adfh(Bg-Ah)+b(3Adfgh\cdots}{\cdots}}{3dfh}$$

$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\big\downarrow$ 131

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(3adfh(Bg-Ah)+\cdots}{\cdots}}{3dfh}$$

$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

3.2. $\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\Big\downarrow 130$$

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{Elliptic}}{}}{3dfh}$$

$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

input
```
Int[((a + b*x)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*b*B*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((2*Sqrt[-(d
*e) + c*f]*(3*A*b*d*f*h + 3*a*B*d*f*h - 2*b*B*(d*f*g + d*e*h + c*f*h))*Sqr
t[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[
c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f
]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]
*(3*a*d*f*h*(B*g - A*h) + b*(3*A*d*f*g*h - B*c*h*(f*g - e*h) - B*d*g*(2*f*
g + e*h)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]
*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f
)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]))/(3*d*f*h
)
```

3.2.3.1 Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2097
```
Int[(((a_.) + (b_.)*(x_))*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_)]*S
qrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*B*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(3*d*f*h)), x] + Simp[1/(3*d*f*h
)   Int[(1/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*a*A*d*f*h -
b*B*(d*e*g + c*f*g + c*e*h) + (3*A*b*d*f*h + B*(3*a*d*f*h - 2*b*(d*f*g + d*
e*h + c*f*h)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

3.2.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2Bb\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(Aa-\frac{2Bb\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\cdots}}}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\cdots}}\right.}{}$
default	Expression too large to display

input
```
int((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2/3*B*b/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*
e*g*x+c*e*g)^(1/2)+2*(A*a-2/3*B*b/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(
g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g
/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*
e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c
/d))^(1/2))+2*(A*b+B*a-2/3*B*b/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/
h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/
(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1
/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d)
)^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(
1/2))))
```

3.2.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 842, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\left(3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}Bbd^2f^2h^2+(2\,Bbd^2f^2g^2+(Bbd^2ef+(Bbcd-3\,(Ba+Ab)d^2)f^2)gh+(2\cdots\right.}{}$$

input
```
integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="fricas")
```

output
```
2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*B*b*d^2*f^2*h^2 + (2*B*b*
d^2*f^2*g^2 + (B*b*d^2*e*f + (B*b*c*d - 3*(B*a + A*b)*d^2)*f^2)*g*h + (2*B
*b*d^2*e^2 + (B*b*c*d - 3*(B*a + A*b)*d^2)*e*f + (2*B*b*c^2 + 9*A*a*d^2 -
3*(B*a + A*b)*c*d)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*
g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^
2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^
2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^
2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e +
 c*f)*h)/(d*f*h)) + 3*(2*B*b*d^2*f^2*g*h + (2*B*b*d^2*e*f + (2*B*b*c*d - 3
*(B*a + A*b)*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f^2*g^2 -
 (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2
), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f -
 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e
*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2
 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h
^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f
 - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d
*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*
f)*h)/(d*f*h))))/(d^3*f^3*h^3)
```

3.2.6 Sympy [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\,(a+bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((A + B*x)*(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)),
x)
```

3.2.7 Maxima [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

3.2.8 Giac [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)
),x)
```

output
```
int(((A + B*x)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)
), x)
```

3.3 $\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.3.1 Optimal result

Integrand size = 33, antiderivative size = 284

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*B*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d/
h/f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(-A*h+B*g)*Elliptic
F(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))
*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)
/d/h/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.3.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(-Bd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx)-iB(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{d}{f}}}{\sqrt{c+dx}}\right.\right.\right.}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{c+}}$$

input
```
Integrate[(A + B*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*(-(B*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x)) - I*B*(d*e - c*f)*h*(
c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
+ d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c
*f*h)/(d*e*h - c*f*h)] + I*d*(B*e - A*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*
x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[S
qrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(d^2*
Sqrt[-c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

3.3.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 176$$

$$\frac{B\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}-\frac{(Bg-Ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \ 124$$

$$\frac{B\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(Bg-Ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{123}$$

$$\frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(Bg-Ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{131}$$

$$\frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}$$

$$\Big\downarrow \text{131}$$

$$\frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{130}$$

$$\frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2(Bg-Ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

input
```
Int[(A + B*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

3.3. $\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*B*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Elli
pticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/
(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h
)]) - (2*Sqrt[-(d*e) + c*f]*(B*g - A*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e +
f*x]*Sqrt[g + h*x])
```

3.3.3.1 Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

3.3.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2A\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2B\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\left(\cdots\right.}{\sqrt{dfh\,x^3+cfh\,x^2+\cdots}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$-\frac{2\left(AF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)de\,h^2-AF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)dfgh-BF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)ce\,h^2+BF\left(\sqrt{-\cdots}\right.\right.}{\cdots}$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2*A*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/
f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(
-g/h+c/d))^(1/2))+2*B*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c
/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g
*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(
g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h
-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

3.3.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(3\sqrt{dfh}Bdfh\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-(d^2ef+cdf^2)gh+(d^2e^2-cdef+c^2f^2)h^2\right)}{3\,d^2f^2h^2},-\frac{4\left(2\,d^3f^3g^3-3\left(d^3ef^2+cd^2f^3\right)g^2h-3\right.}{}\right.\right.}{}$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
fricas")
```

output
```
-2/3*(3*sqrt(d*f*h)*B*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f +
c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*
d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f
^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3
*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f
+ c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(
2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e
*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c
^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h
))) + (B*d*f*g + (B*d*e + (B*c - 3*A*d)*f)*h)*sqrt(d*f*h)*weierstrassPInve
rse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*
f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*
g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c
*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x
 + d*f*g + (d*e + c*f)*h)/(d*f*h)))/(d^2*f^2*h^2)
```

3.3.6 Sympy [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.3.7 Maxima [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((B*x + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.3.8 Giac [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
giac")
```

output
```
integrate((B*x + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.4 $\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.4.1 Optimal result

Integrand size = 40, antiderivative size = 313

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\left(A-\frac{aB}{b}\right)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*B*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h
+d*g))^(1/2)/b/d/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(A-a*B/b)*EllipticP
i(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+
d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(
d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.4.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.95 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\frac{2i\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(b(-Bc+Ad)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right),\frac{dfg-cfh}{deh-cfh}\right)+(-Ab+aB)d\,\text{EllipticPi}\left(\right.\right.}{b(-bc+ad)\sqrt{-c+\frac{de}{f}}f\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{g+hx}}$$

input
```
Integrate[(A + B*x)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
((2*I)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*(b*(-(B*c) + A*d)*E
llipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e
*h - c*f*h)] + (-(A*b) + a*B)*d*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*
f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h -
 c*f*h)]))/(b*(-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*f*Sqrt[(d*(e + f*x))/(f*(c
 + d*x))]*Sqrt[g + h*x])
```

3.4.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {2110, 27, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$\Big\downarrow \text{ 2110}$$
$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\int \frac{B}{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$
$$\Big\downarrow \text{ 27}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{B\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}$$

$$\Big\downarrow \text{131}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{B\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b\sqrt{e+fx}}$$

$$\Big\downarrow \text{131}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{B\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{130}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{187}$$

$$\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-2\left(A-\frac{aB}{b}\right)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{413}$$

$$\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\left(A-\frac{aB}{b}\right)\sqrt{\frac{f(c+dx)}{de-cf}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\left(A-\frac{aB}{b}\right)\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

3.4. $\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\downarrow 412$$

$$\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\left(A-\frac{aB}{b}\right)\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\operatorname{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

input
```
Int[(A + B*x)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*B*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))
/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]]
, ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*d*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*
x]) - (2*(A - (a*B)/b)*Sqrt[-(d*e) + c*f]*Sqrt[1 + (f*(c + d*x))/(d*e - c*
f)]*Sqrt[1 + (h*(c + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c
 - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c
*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d
*x))/d]*Sqrt[g - (c*h)/d + (h*(c + d*x))/d])
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.53

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2B\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2(Ab-Ba)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+}{-\frac{g}{h}}}}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+c}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$-\frac{2\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{-\frac{(hx+g)f}{eh-fg}}\,\sqrt{\frac{(dx+c)h}{ch-dg}}\,\sqrt{\frac{(fx+e)h}{eh-fg}}\left(A\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(eh-fg)b}{f(ah-gb)},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)be\,h^2-A\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\right.\right.}{}$

input
```
int((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2*B/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+
e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*
f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)
/(-g/h+c/d))^(1/2))+2*(A*b-B*a)/b^2*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((
x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d
*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*Ellipti
cPi(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d)
)^(1/2)))
```

3.4.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

3.4.6 Sympy [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+Bx}{(a+bx)\,\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((A + B*x)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)),
x)
```

3.4.7 Maxima [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((B*x + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

3.4.8 Giac [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((B*x + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),
x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),
 x)
```

3.5 $\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.5.1 Optimal result

Integrand size = 40, antiderivative size = 678

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$+\frac{\sqrt{-de+cf}\,(3a^2Abdfh-a^3Bdfh-b^3(2Bceg-A(deg+cfg+ceh))+ab^2(B(deg+cfg+ceh)-2A(}{b(bc-ad)^2\sqrt{f}(be-af)(bg-ah}$$

output
```
-b*(A*b-B*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*
e)/(-a*h+b*g)/(b*x+a)+(A*b-B*a)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^
(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(c*f-d*e)^(1/2)*(d*(f*x+e
)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(f*x+e)
^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+(3*a^2*A*b*d*f*h-a^3*B*d*f*h-b^3*(2*B*
c*e*g-A*(c*e*h+c*f*g+d*e*g))+a*b^2*(B*(c*e*h+c*f*g+d*e*g)-2*A*(c*f*h+d*e*h
+d*f*g)))*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(
-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/
(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)/(
-a*h+b*g)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-(A*b-B*a)*EllipticF(f^(1/2)*
(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(
c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b
/(-a*d+b*c)/(-a*f+b*e)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.5.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.13 (sec) , antiderivative size = 3412, normalized size of antiderivative = 5.03

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
-((b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(
b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((c + d*x)^(3/2)*(A*b^3*c*Sqrt[-c + (
d*e)/f]*f*h - a*b^2*B*c*Sqrt[-c + (d*e)/f]*f*h - a*A*b^2*d*Sqrt[-c + (d*e)
/f]*f*h + a^2*b*B*d*Sqrt[-c + (d*e)/f]*f*h + (A*b^3*c*d^2*e*Sqrt[-c + (d*e
)/f]*g)/(c + d*x)^2 - (a*b^2*B*c*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 -
 (a*A*b^2*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 + (a^2*b*B*d^3*e*Sqrt[-c
 + (d*e)/f]*g)/(c + d*x)^2 - (A*b^3*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x
)^2 + (a*b^2*B*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a*A*b^2*c*d^2*
Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 - (a^2*b*B*c*d^2*Sqrt[-c + (d*e)/f]*f*
g)/(c + d*x)^2 - (A*b^3*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*b^2
*B*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*A*b^2*c*d^2*e*Sqrt[-c +
(d*e)/f]*h)/(c + d*x)^2 - (a^2*b*B*c*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)
^2 + (A*b^3*c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (a*b^2*B*c^3*Sqrt[-c
 + (d*e)/f]*f*h)/(c + d*x)^2 - (a*A*b^2*c^2*d*Sqrt[-c + (d*e)/f]*f*h)/(c +
 d*x)^2 + (a^2*b*B*c^2*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (A*b^3*c*d*
Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (a*b^2*B*c*d*Sqrt[-c + (d*e)/f]*f*g)/(
c + d*x) - (a*A*b^2*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (a^2*b*B*d^2*S
qrt[-c + (d*e)/f]*f*g)/(c + d*x) + (A*b^3*c*d*e*Sqrt[-c + (d*e)/f]*h)/(c +
 d*x) - (a*b^2*B*c*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (a*A*b^2*d^2*e*Sq
rt[-c + (d*e)/f]*h)/(c + d*x) + (a^2*b*B*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c...
```

3.5.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2102, 25, 2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int -\frac{2Adfha^2+b(B(deg+cfg+ceh)-2A(dfg+deh+cfh))a-2(Ab-aB)dfhxa-b(Ab-aB)dfhx^2-b^2(2Bceg-A(deg+cfg+ceh))}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2(bc-ad)(be-af)(bg-ah)}-\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{2Adfha^2+b(B(deg+cfg+ceh)-2A(dfg+deh+cfh))a-2(Ab-aB)dfhxa-b(Ab-aB)dfhx^2-b^2(2Bceg-A(deg+cfg+ceh))}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2(bc-ad)(be-af)(bg-ah)}-\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 2110

$$-\frac{\int \frac{\frac{Bdfha^2}{b}-Adfha+(aBdfh-Abdfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+c}{b}}{2(bc-ad)(be-af)(bg-ah)}$$
$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 176

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df(A}{}}{2(bc-ad)(be-af)(bg-ah)}$$
$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 124

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df(A}{}}{2(bc-ad)(be-af)(bg-ah)}$$
$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 123

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df(A}{}}{2(bc-ad)(be-af)(bg-}$$
$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 131

3.5. $\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df(A}{}}{2(bc-ad)(be-}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 131

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df(A}{}}{2(bc-a}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 130

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2\sqrt{f}}{}}{2(bc-}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 187

$$-\frac{-\frac{2(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{}}}{b}}{}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 413

$$-\frac{-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{}{(bc-ad-b(c+dx))\sqrt{\frac{f(}{d}}}}{b\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}}{}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 413

3.5. $\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)\int \frac{}{(bc-ad}}{b\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}}{}$$

$$-\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow 412$$

$$-\frac{-\frac{2\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)}{b\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}}{}$$

$$-\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

input
```
Int[(A + B*x)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
-((b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(
b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((-2*(A*b - a*B)*Sqrt[f]*Sqrt[-(d*e)
+ c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqr
t[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])
/(Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) + (2*(A*b - a*B)*Sqrt[f]*
Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g
+ h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e)
+ c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt[e + f*x]*Sqrt[g + h*x])
 - (2*Sqrt[-(d*e) + c*f]*(3*a^2*A*b*d*f*h - a^3*B*d*f*h - b^3*(2*B*c*e*g -
 A*(d*e*g + c*f*g + c*e*h)) + a*b^2*(B*(d*e*g + c*f*g + c*e*h) - 2*A*(d*f*
g + d*e*h + c*f*h)))*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1 + (h*(c +
d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(
Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h)
)])/(b*(b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sqrt[g - (c
*h)/d + (h*(c + d*x))/d]))/(2*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))
```

3.5. $\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && !GtQ[c, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

3.5. $\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.5.4 Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1208
default	Expression too large to display	13344

input
```
int((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+
a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+
c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)-a*d*f*h*(A*b-B*a)/(a^3*d*f*h-
a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^
3*c*e*g)/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*
((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*
x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+
e/f)/(-g/h+c/d))^(1/2))-d*f*h*(A*b-B*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h
-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(g/h-e/f)*((x+
g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2
)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^
(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/
d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))
^(1/2)))+(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f*g+
A*b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g-B*a^3*d*f*h+B*a*b^2*c*e*h+B*a*b^2*c*f*
g+B*a*b^2*d*e*g-2*B*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*
f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^2*(g/h-e/f)*((x+g/h)/
(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*
f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1...
```

3.5.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="fricas")
```

output
```
Timed out
```

3.5.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),
x)
```

output
```
Timed out
```

3.5.7 Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*x + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

3.5.8 Giac [F]

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((B*x + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)^2\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)
),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)
), x)
```

3.6 $\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.6.1 Optimal result

Integrand size = 42, antiderivative size = 981

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{(5aBdfh+b(4Adfh-3B(dfg+deh+cfh)))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}}$$

$$+\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(5aBdfh+b(4Adfh-3B(dfg+deh+cfh)))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{d}}{\sqrt{f}}\right.\right.}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$-\frac{(be-af)\sqrt{bg-ah}(3aBdfh+b(4Adfh-B(cfh+3d(fg+eh))))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\text{arcs}\right.}{4bdf^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$+\frac{\sqrt{-dg+ch}(4dfh(2a(2Ab+aB)dfh-bB(b(deg+cfg+ceh)+a(dfg+deh+cfh)))-(adfh+b(dfg+}{}$$

output
```
1/4*(4*d*f*h*(2*a*(2*A*b+B*a)*d*f*h-b*B*(b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*
e*h+d*f*g)))-(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*(5*a*B*d*f*h+b*(4*A*d*f*h-3*B
*(c*f*h+d*e*h+d*f*g))))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/
(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h
+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-
c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b/d^
2/f^2/h^3/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/4*(5*a*B*d*f*h+b*
(4*A*d*f*h-3*B*(c*f*h+d*e*h+d*f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(
1/2)/d/f^2/h^2/(d*x+c)^(1/2)+1/2*b*B*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(
1/2)*(h*x+g)^(1/2)/d/f/h-1/4*(-a*f+b*e)*(3*a*B*d*f*h+b*(4*A*d*f*h-B*(c*f*h
+3*d*(e*h+f*g))))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2
)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*
h+b*g)^(1/2)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/b
/d/f^2/h^2/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/
(b*x+a))^(1/2)-1/4*(5*a*B*d*f*h+b*(4*A*d*f*h-3*B*(c*f*h+d*e*h+d*f*g)))*Ell
ipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*
d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g
)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d^2/f
^2/h^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

3.6.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21961 vs. $2(981) = 1962$.

Time = 36.59 (sec) , antiderivative size = 21961, normalized size of antiderivative = 22.39

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
Result too large to show
```

3.6.3 Rubi [A] (warning: unable to verify)

Time = 2.96 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {2100, 2105, 25, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 2100$$

$$\frac{\int \frac{4Adfha^2+b(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))x^2-bB(bceg+a(deg+cfg+ceh))+2(2Bdfha^2+4Abdfha-bB(dfg+deh+cfh)a-b^2B(deg+}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{4dfh}$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 2105$$

$$\frac{\int -\frac{b\left((bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg\right)\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{b\left((bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(df\right)\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{(bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg+\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 194$$

$$-\frac{\int \frac{(bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg+\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}\cdots$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 327

$$-\frac{\int \frac{(bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg+\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}\cdots$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 2101

$$-\frac{\frac{\left((adfh+b(cfh+deh+dfg))(5aBdfh+4Abdfh-3bB(cfh+deh+dfg))-4dfh\left(2a^2Bdfh+4aAbdfh-abB(cfh+deh+dfg)-b^2B(ceh+cfg+deg)\right)\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g\cdots}}}{b}}{2dfh}\cdots$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 183

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{\sqrt{a+bx}\sqrt{\cdots}}{\cdots}$$

$\downarrow$ 188

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{\sqrt{a+bx}\sqrt{\cdots}}{\cdots}$$

$\downarrow$ 321

3.6. $\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{\sqrt{a+bx}\sqrt{e}}{}$$

$\downarrow$ 412

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{\sqrt{a+bx}\sqrt{e}}{}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(b*B*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h) +
(((4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x
]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (Sqrt[d*g - c*h]*Sqrt
[f*g - e*h]*(4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h))*Sq
rt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*Ellip
ticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x]
)], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(d*f*h*Sqrt[((d*
e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - ((2*d*(b*e -
 a*f)*Sqrt[b*g - a*h]*(4*A*b*d*f*h + 3*a*B*d*f*h - b*B*(c*f*h + 3*d*(f*g +
 e*h)))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x
]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a
 + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*Sqr
t[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a
+ b*x)))]) + (2*Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*
(4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h)) - 4*d*f*h*(4*a
*A*b*d*f*h + 2*a^2*B*d*f*h - b^2*B*(d*e*g + c*f*g + c*e*h) - a*b*B*(d*f*g
+ d*e*h + c*f*h)))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
+ b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[
-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])
/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c -...
```

3.6. $\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.6. $\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1813 vs. $2(898) = 1796$.

Time = 5.18 (sec) , antiderivative size = 1814, normalized size of antiderivative = 1.85

method	result	size
elliptic	Expression too large to display	1814
default	Expression too large to display	55936

input
```
int((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(1/2*B*b/d/f/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+
b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*
c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/
2)+2*(a^2*A-1/2*B*b/d/f/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g
))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/
d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x
+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h
))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d
)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(2*a*b*A+a^2*B-1/2*B*b/d/f/h*(
a*c*f*h+a*d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+c/d)*(x
+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(
x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c
/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((
-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f
)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(
x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+
g/h))^(1/2)))+(b^2*A+2*a*b*B-1/2*B*b/d/f/h*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*
d*e*h+3/2*b*d*f*g))*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)
/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+...
```

3.6.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
Timed out
```

3.6.6 Sympy [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\,(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Integral((A + B*x)*(a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g +
h*x)), x)
```

3.6.7 Maxima [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.6.8 Giac [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)
^(1/2)),x)
```

output
```
int(((A + B*x)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)
^(1/2)), x)
```

3.7 $\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.7.1 Optimal result

Integrand size = 42, antiderivative size = 736

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}} - \frac{B\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{B(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{\sqrt{-dg+ch}(2Abdfh+B(adfh-b(dfg+deh+cfh)))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}}{bd\sqrt{bc-ad}fh^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
(2*A*b*d*f*h+B*(a*d*f*h-b*(c*f*h+d*e*h+d*f*g)))*(b*x+a)*EllipticPi((-a*d+b
*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+
b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2
)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+
f*g)/(b*x+a))^(1/2)/b/d/f/h^2/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
+B*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/h/(d*x+c)^(1/2)-B*(-a*f+b*e
)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),
(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^(1/2)*((-
a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/b/f/h/(-e*h+f*g)^
(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-B*Ellip
ticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+
b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^
(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d/f/h/(
(-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

3.7.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8030 vs. 2(736) = 1472.

Time = 42.32 (sec) , antiderivative size = 8030, normalized size of antiderivative = 10.91

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
Result too large to show
```

3.7.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 725, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2099, 183, 188, 194, 321, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{ 2099}$$

$$\frac{1}{2}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{B(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{B(be-af)(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bfh}+\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 183}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{B(be-af)(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bfh}+\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 188}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{B\sqrt{g+hx}(be-af)(bg-ah)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{bfh\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 194}$$

3.7. $\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int\frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B\sqrt{g+hx}(be-af)(bg-ah)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int\frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{bfh\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{B\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int\frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$\downarrow$ 321

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int\frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int\frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}-$$

$$\frac{B\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+$$

$$\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$\downarrow$ 327

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int\frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{B\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$\downarrow$ 412

3.7. $\int\frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}}{\sqrt{ch-dg}}\right.\right.}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

$$\frac{B\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{B\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(B*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (B*Sqr
t[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/
((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])
/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*
(d*g - c*h))])/(d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))
]*Sqrt[g + h*x]) - (B*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d
*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a
*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g -
 e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*f*h*Sqrt[f*g - e*h]*Sqrt[c + d*x]*S
qrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + ((2*A + B*(a/b
- c/d - e/f - g/h))*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*
x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a
+ b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c -
 a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*
g - c*h))/((b*c - a*d)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqr
t[e + f*x])
```

3.7. $\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.7.3.1 Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2099
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[B*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*Sqrt[c + d*x])), x] + (-Sim
p[B*(b*e - a*f)*((b*g - a*h)/(2*b*f*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x
]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Simp[B*(d*e - c*f)*((d*g - c*h)/(2
*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Simp[(2*A*b*d*f*h + B*(a*d*f*h - b*(d*f*g + d*e*h + c*f*h)))/(2*b
*d*f*h)   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && NeQ[2*A*d*f - B*(d*e +
 c*f), 0]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. $2(671) = 1342$.

Time = 5.15 (sec) , antiderivative size = 1544, normalized size of antiderivative = 2.10

method	result	size
elliptic	Expression too large to display	1544
default	Expression too large to display	21369

input
```
int((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*A*a*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c
/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+
a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a
/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b
)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(A*b
+B*a)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((
-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)
/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+
g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),
((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((
-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/
d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+B*b*((x+a/b)*(x+e/f)*(x+g/h)+(
g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/
b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d
))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*Elliptic
F(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b
+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/
b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d
+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi((...
```

3.7.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
Timed out
```

3.7.6 Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x
)), x)
```

3.7.7 Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

3.7.8 Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)
^(1/2)),x)
```

output
```
int(((A + B*x)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)
^(1/2)), x)
```

3.8 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.8.1 Optimal result

Integrand size = 42, antiderivative size = 442

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(Ab-aB)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{2B\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-a}{(bc-a}\right.}{b\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2*B*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x
+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e
*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1
/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b/h/(-a*d+b*c)^(1/2)/(d*
x+c)^(1/2)/(f*x+e)^(1/2)+2*(A*b-B*a)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1
/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*
h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)
/b/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-
e*h+f*g)/(b*x+a))^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 24.66 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2(a+bx)^{3/2}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-\frac{Ab\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right),\frac{(-bc+ad)(-fg+eh)}{(be-af)(dg-ch)}\right)}{(bg-ah)(a+bx)\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}} - \frac{aB\sqrt{\frac{(bg-a}{(fg-e}}}{}\right.}{}$$

input
```
Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
(2*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(
-((A*b*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*Ell
ipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], (
(-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*(a
 + b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])) - (a*B*
Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*EllipticF[
ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c)
 + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((-(b*g) + a*h)*(a + b
*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]) + (B*(-(f*g)
 + e*h)*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^
2*(a + b*x)^2))]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a*f)*h), ArcSin[Sqr
t[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-
(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*e - a*f)*h)))/(b*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

3.8.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 2101$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{B\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}$$

$$\Big\downarrow \ 183$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2B(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \ 188$$

$$\frac{2\sqrt{g+hx}(Ab-aB)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2B(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \ 321$$

$$\frac{2B(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{g+hx}(Ab-aB)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \ 412$$

$$\frac{2\sqrt{g+hx}(Ab-aB)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2B(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{bh\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

input
```
Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

3.8. $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*(A*b - a*B)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[
g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]
*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])
/(b*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g +
 h*x))/((f*g - e*h)*(a + b*x)))]) + (2*B*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt
[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f
*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)
), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x
])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*Sqrt[b*c - a
*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.8.3.1 Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

3.8.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(404) = 808$.

Time = 6.53 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.92

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\left(\frac{2A\left(\frac{g}{h}-\frac{a}{b}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^{2}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}F\left(\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}},\right.}{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*A*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d
))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/
b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b
)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/
(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*B*(g/h
-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*
(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^
(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2
)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)
*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*
(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/
b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))
```

3.8.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
Timed out
```

3.8.6 Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x
)), x)
```

3.8.7 Maxima [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

3.8.8 Giac [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(
1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(
1/2)), x)
```

3.9 $\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.9.1 Optimal result

Integrand size = 42, antiderivative size = 606

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(Ab-aB)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}$$
$$-\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$
$$-\frac{2(Ab-aB)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+\frac{2(Bc-Ad)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

output
```
2*(A*b-B*a)*d*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b
*e)/(-a*h+b*g)/(d*x+c)^(1/2)-2*b*(A*b-B*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*
x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*(-A*d+B*c)*Ell
ipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a
*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f
+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*h+b*g)^(1/2)/(-e*h+f*g)^
(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-2*(A*b-
B*a)*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/
2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(
-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/
2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c)
)^(1/2)/(h*x+g)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 26.02 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left((Ab-aB)(\right.}{(b}$$

input
```
Integrate[(A + B*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
(2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
f*x)^(3/2)*(g + h*x)^(3/2)*((A*b - a*B)*(d*g - c*h)*EllipticE[ArcSin[Sqrt[
((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g -
e*h))/((b*e - a*f)*(d*g - c*h))] + (B*c - A*d)*(b*g - a*h)*EllipticF[ArcSi
n[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*
(f*g - e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*g - e*h)^3*(a +
b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/
((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

3.9.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 2102$$

$$\frac{\int \frac{-Adfha^2-b(B(deg+cfg+ceh)-A(dfg+deh+cfh))a+2b(Ab-aB)dfhx^2+b^2Bceg+(Ab-aB)(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 2105$$

$$\frac{(Ab-aB)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{\int \frac{2bd(Bc-Ad)f(be-af)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB}{\sqrt{c+dx}}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 27

$$\frac{(Ab-aB)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+(be-af)(bg-ah)(Bc-Ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 188

$$\frac{(Ab-aB)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2\sqrt{g+hx}(be-af)(bg-ah)(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}}}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 194

$$\frac{\frac{2\sqrt{g+hx}(be-af)(bg-ah)(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2\sqrt{a+bx}(Ab-aB)(dg-ch)\sqrt{-\frac{(g+hx)(de}{(c+dx)(fg}}}{\sqrt{g+hx}\sqrt{\frac{(a+b}{(c+d}}}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 321

$$\frac{-\frac{2\sqrt{a+bx}(Ab-aB)(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{2\sqrt{g+hx}(be-af)\sqrt{bg-ah}(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\Big(}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(}{(a+bx)(}}}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 327

3.9. $\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{\frac{2\sqrt{g+hx}(be-af)\sqrt{bg-ah}(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}} - \frac{2\sqrt{a+bx}(Ab-aB)\sqrt{dg-ch}\sqrt{fg-eh}}{}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

input
```
Int[(A + B*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(-2*b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*
(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*(A*b - a*B)*d*Sqrt[a + b*x]*S
qrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*(A*b - a*B)*Sqrt[d*g - c*h]
*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)
*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g -
 e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))
])/(Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) +
 (2*(B*c - A*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sq
rt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))
/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e
- a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]))/((b*c - a*d)*(b*e - a*f)*(b*
g - a*h))
```

3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.9. $\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2249 vs. $2(552) = 1104$.

Time = 7.69 (sec) , antiderivative size = 2250, normalized size of antiderivative = 3.71

method	result	size
elliptic	Expression too large to display	2250
default	Expression too large to display	18724

input
```
int((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+
b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-
a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a)/((x+a
/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b
*d*e*g*x+b*c*e*g))^(1/2)+2*(B/b+1/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f
*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(A*b-B*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d
*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b
*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h
+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a))*(g/h-a/b)*((-g/h+c/d)*(x+a/
b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c
/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+
a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)
*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g
/h))^(1/2))+2*(-(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(A*b-B*a)/(a^3*d*f*h-a^2
*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c
*e*g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a
^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a))*(g/h-
a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(
x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d)...
```

3.9.5 Fricas [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
integral((B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
/(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2*a*b*d)*f)
*h)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)*e + (2*a
*b*c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)*f)*g +
 (a^2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (2*a*b*c
 + a^2*d)*e)*g)*x), x)
```

3.9.6 Sympy [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+Bx}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Integral((A + B*x)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g +
h*x)), x)
```

3.9.7 Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.9.8 Giac [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(
1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(
1/2)), x)
```

3.10 $\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.10.1 Optimal result

Integrand size = 42, antiderivative size = 1081

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d(3a^3Bdfh+b^3(3Bceg-2A(deg+cfg+ceh))-ab^2(B(de\cdots}{\cdots}$$

$$-\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2b(3a^3Bdfh+b^3(3Bceg-2A(deg+cfg+ceh))-ab^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))-\cdots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}(3a^3Bdfh+b^3(3Bceg-2A(deg+cfg+ceh))-ab^2(B(deg+cfg+ceh)-4A(dfg\cdots}{3(bc-ad)^2(be-af\cdots}$$

$$-\frac{2(3a^2d(Bc-Ad)fh+b^2(3Bcdeg-A(2d^2eg-c^2fh+cd(fg+eh)))+ab(3Ad^2(fg+eh)-B(d^2eg+c^2\cdots}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{fg\cdots}}$$

output
```
2/3*d*(3*a^3*B*d*f*h+b^3*(3*B*c*e*g-2*A*(c*e*h+c*f*g+d*e*g))-a*b^2*(B*(c*e
*h+c*f*g+d*e*g)-4*A*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*A*d*f*h+B*(c*f*h+d*e*h+d
*f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^
2/(-a*h+b*g)^2/(d*x+c)^(1/2)-2/3*b*(A*b-B*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(
h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)-2/3*b*(3*a^3*B
*d*f*h+b^3*(3*B*c*e*g-2*A*(c*e*h+c*f*g+d*e*g))-a*b^2*(B*(c*e*h+c*f*g+d*e*g
)-4*A*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*A*d*f*h+B*(c*f*h+d*e*h+d*f*g)))*(d*x+c
)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2
/(b*x+a)^(1/2)-2/3*(3*a^2*d*(-A*d+B*c)*f*h+b^2*(3*B*c*d*e*g-A*(2*d^2*e*g-c
^2*f*h+c*d*(e*h+f*g)))+a*b*(3*A*d^2*(e*h+f*g)-B*(d^2*e*g+c^2*f*h+2*c*d*(e*
h+f*g))))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a
)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*
(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-
a*h+b*g)^(3/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f
*g)/(b*x+a))^(1/2)-2/3*(3*a^3*B*d*f*h+b^3*(3*B*c*e*g-2*A*(c*e*h+c*f*g+d*e*
g))-a*b^2*(B*(c*e*h+c*f*g+d*e*g)-4*A*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*A*d*f*h
+B*(c*f*h+d*e*h+d*f*g)))*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*
g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2)
)*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e
*h+f*g)/(d*x+c))^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/((-c*f+d*...
```

3.10.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10828 vs. $2(1081) = 2162$.

Time = 39.61 (sec) , antiderivative size = 10828, normalized size of antiderivative = 10.02

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
Result too large to show
```

3.10.3 Rubi [A] (verified)

Time = 3.68 (sec) , antiderivative size = 1068, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2102, 25, 2102, 25, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int -\frac{3Adfha^2+b(B(deg+cfg+ceh)-3A(dfg+deh+cfh))a-b^2(3Bceg-2A(deg+cfg+ceh))-(Ab-aB)(3adfh-b(dfg+deh+cfh))x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3(bc-ad)(be-af)(bg-ah)} - \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{3Adfha^2+b(B(deg+cfg+ceh)-3A(dfg+deh+cfh))a-b^2(3Bceg-2A(deg+cfg+ceh))-(Ab-aB)(3adfh-b(dfg+deh+cfh))x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3(bc-ad)(be-af)(bg-ah)} - \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2102}$$

$$-\frac{\int -\frac{2bdfh\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)x^2+(adfh+b(dfg+deh+cfh}{}}{}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(3a^3Bdfh-a^2b(6Adfh+B(cfh+deh+dfg))-ab^2(B(ceh+cfg+deg)-4A(cfh+deh+dfg))+b^3(3Bceg-2A(ceh+cfg+d}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}}{}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2105}$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(3a^3Bdfh-a^2b(6Adfh+B(cfh+deh+dfg))-ab^2(B(ceh+cfg+deg)-4A(cfh+deh+dfg))+b^3(3Bceg-2A(ceh+cfg+d}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 188

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$\downarrow$ 194

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$\downarrow$ 321

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$\downarrow$ 327

3.10. $\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(-2*b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) - ((2*b*(3*a^3*B*d*f*h + b^3*(3
*B*c*e*g - 2*A*(d*e*g + c*f*g + c*e*h)) - a*b^2*(B*(d*e*g + c*f*g + c*e*h)
 - 4*A*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*A*d*f*h + B*(d*f*g + d*e*h + c*
f*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)
*(b*g - a*h)*Sqrt[a + b*x]) - ((2*d*(3*a^3*B*d*f*h + b^3*(3*B*c*e*g - 2*A*
(d*e*g + c*f*g + c*e*h)) - a*b^2*(B*(d*e*g + c*f*g + c*e*h) - 4*A*(d*f*g +
 d*e*h + c*f*h)) - a^2*b*(6*A*d*f*h + B*(d*f*g + d*e*h + c*f*h)))*Sqrt[a +
 b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*Sqrt[d*g - c*h]*Sqrt
[f*g - e*h]*(3*a^3*B*d*f*h + b^3*(3*B*c*e*g - 2*A*(d*e*g + c*f*g + c*e*h))
 - a*b^2*(B*(d*e*g + c*f*g + c*e*h) - 4*A*(d*f*g + d*e*h + c*f*h)) - a^2*b
*(6*A*d*f*h + B*(d*f*g + d*e*h + c*f*h)))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f
)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*S
qrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/
((b*e - a*f)*(d*g - c*h))]])/(Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c
+ d*x))]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[b*g - a*h]*(3*a^2*d*(B*c - A
*d)*f*h + b^2*(3*B*c*d*e*g - 2*A*d^2*e*g + A*c^2*f*h - A*c*d*(f*g + e*h))
+ a*b*(3*A*d^2*(f*g + e*h) - B*(d^2*e*g + c^2*f*h + 2*c*d*(f*g + e*h))))*S
qrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Ellipti
cF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x...
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

3.10. $\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3388 vs. $2(1009) = 2018$.

Time = 10.18 (sec) , antiderivative size = 3389, normalized size of antiderivative = 3.14

method	result	size
elliptic	Expression too large to display	3389
default	Expression too large to display	104801

input
```
int((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2/3/b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*
g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a)*(b*d*f*h*x^4+a*
d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*
f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+
b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+2/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^
2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*
h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2
*(6*A*a^2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*f*g+2*A*b^3*
c*e*h+2*A*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*f*h+B*a^2*b*d*e*
h+B*a^2*b*d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-3*B*b^3*c*e*g)/(
(x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g
*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(-1/3*(3*A*a*b*d*f*h-A*b^2*c*f*h-A*b^2*d*e*
h-A*b^2*d*f*g-3*B*a^2*d*f*h+B*a*b*c*f*h+B*a*b*d*e*h+B*a*b*d*f*g)/b/(a^3*d*
f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*
g-b^3*c*e*g)+1/3/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*
c*f*g+b^2*d*e*g)*(6*A*a^2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^
2*d*f*g+2*A*b^3*c*e*h+2*A*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*
f*h+B*a^2*b*d*e*h+B*a^2*b*d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-
3*B*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e...
```

3.10.5 Fricas [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
integral((B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3*a*b^2*d)*
f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*b^2*d)*e
+ 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a*b^2*c +
 a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*h)*x^3
+ ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f + (3*a^2
*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3*d)*e)*g
)*x), x)
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Timed out
```

3.10.7 Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.10.8 Giac [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(
1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(
1/2)), x)
```

3.11 $\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.11.1 Optimal result

Integrand size = 49, antiderivative size = 898

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{2fh^2\sqrt{c+dx}}$$
$$+\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$
$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(5adfh-b(3dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-}{(be-}\right.}{2dfh^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{(be-af)\sqrt{bg-ah}(3adfh+b(cfh-d(3fg+eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+f}}{\sqrt{fg-eh}\sqrt{a+b}}\right.\right.}{2bfh^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$-\frac{\sqrt{-dg+ch}(6abd^2f^2gh-3a^2d^2f^2h^2+b^2(2cdefh^2-c^2f^2h^2-d^2(3f^2g^2+e^2h^2)))\,(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}}{2bd\sqrt{bc-ad}fh^3\sqrt{c+dx}\sqrt{e+f}}$$

output
```
-1/2*(6*a*b*d^2*f^2*g*h-3*a^2*d^2*f^2*h^2+b^2*(2*c*d*e*f*h^2-c^2*f^2*h^2-d
^2*(e^2*h^2+3*f^2*g^2)))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)
/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*
h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(
-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b/d
/f/h^3/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/2*(5*a*d*f*h-b*(c*f*
h+d*e*h+3*d*f*g))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/h^2/(d*x+c)^
(1/2)+b*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/h-1/2*(-a*
f+b*e)*(3*a*d*f*h+b*(c*f*h-d*(e*h+3*f*g)))*EllipticF((-a*h+b*g)^(1/2)*(f*x
+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e
)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^(1/2)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+
a))^(1/2)*(h*x+g)^(1/2)/b/f/h^2/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e
)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-1/2*(5*a*d*f*h-b*(c*f*h+d*e*h+3*d*f*g)
)*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),
((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*
h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/
d/f/h^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

3.11.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 15131 vs. $2(898) = 1796$.

Time = 35.44 (sec) , antiderivative size = 15131, normalized size of antiderivative = 16.85

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^(3/2)*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e +
f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

3.11.3 Rubi [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 892, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.224, Rules used = {2100, 27, 2105, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 2100$$

$$\frac{\int \frac{2\left(bdf(5adfh-b(3dfg+deh+cfh))x^2+2df\left(2dfha^2-b(dfg-deh-cfh)a-b^2(deg+cfg+ceh)\right)x+df\left(2a^2(de+cf)h-b(bceg+a(deg+cfg+ceh))\right)\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{4dfh}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{bdf(5adfh-b(3dfg+deh+cfh))x^2+2df\left(2dfha^2-b(dfg-deh-cfh)a-b^2(deg+cfg+ceh)\right)x+df\left(2a^2(de+cf)h-b(bceg+a(deg+cfg+ceh))\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2dfh}+$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\Big\downarrow \ 2105$$

$$\frac{\frac{\int \frac{bdf\left(deg(3dfg+deh-cfh)b^2-afh\left(-fhc^2-d(fg-eh)c+7d^2eg\right)b+a^2df(4de-cf)h^2-\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}}{2dfh}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{deg(3dfg+deh-cfh)b^2-afh\left(-fhc^2-d(fg-eh)c+7d^2eg\right)b+a^2df(4de-cf)h^2-\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2h}}{2dfh}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\Big\downarrow \ 194$$

$$\frac{\frac{\int \frac{deg(3dfg+deh-cfh)b^2-afh\left(-fhc^2-d(fg-eh)c+7d^2eg\right)b+a^2df(4de-cf)h^2-\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2h}}{\;}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\downarrow 327$$

$$\frac{\frac{\int \frac{deg(3dfg+deh-cfh)b^2-afh\left(-fhc^2-d(fg-eh)c+7d^2eg\right)b+a^2df(4de-cf)h^2-\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2h}}{\;}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\downarrow 2101$$

$$\frac{-\frac{\left(-3a^2d^2f^2h^2+6abd^2f^2gh+b^2\left(-c^2f^2h^2+2cdefh^2-\left(d^2\left(e^2h^2+3f^2g^2\right)\right)\right)\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{d(be-af)(bg-ah)(3adfh-b(-cfh+deh+3dfg))\int \frac{}{\sqrt{a\cdots}}}{b}}{2h}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\downarrow 183$$

$$\frac{-\frac{2(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(-3a^2d^2f^2h^2+6abd^2f^2gh+b^2\left(-c^2f^2h^2+2cdefh^2-\left(d^2\left(e^2h^2+3f^2g^2\right)\right)\right)\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+\cdots}}}{b\sqrt{c+dx}\sqrt{e+fx}}}{2h}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\downarrow 188$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b}{h}+$$

$$\frac{-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(5adfh-b(3dfg+deh+cfh))}{h\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+h\cdots}}{h}}{\;}$$

$$\downarrow 321$$

3.11. $\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b}{h}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(5adfh-b(3dfg+deh+cfh))}{h\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+h\cdots}}{h\cdots}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b}{h}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(5adfh-b(3dfg+deh+cfh))}{h\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+h\cdots}}{h\cdots}$$

input
```
Int[((a + b*x)^(3/2)*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*S
qrt[g + h*x]),x]
```

output
```
(b*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/h + ((d*(5*a*d
*f*h - b*(3*d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h
*x])/(h*Sqrt[c + d*x]) - (Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(5*a*d*f*h - b*(
3*d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f
*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(S
qrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*
g - c*h))])/(h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[
g + h*x]) + ((-2*d*(b*e - a*f)*Sqrt[b*g - a*h]*(3*a*d*f*h - b*(3*d*f*g + d
*e*h - c*f*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[
g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]
*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])
/(b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e
*h)*(a + b*x)))]) - (2*Sqrt[-(d*g) + c*h]*(6*a*b*d^2*f^2*g*h - 3*a^2*d^2*f
^2*h^2 + b^2*(2*c*d*e*f*h^2 - c^2*f^2*h^2 - d^2*(3*f^2*g^2 + e^2*h^2)))*(a
 + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g -
 a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b
*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*
Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*
Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x]))/(2*h))/(2*d*f*h)
```

3.11. $\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.11. $\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.11.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1808 vs. 2(817) = 1634.

Time = 5.17 (sec) , antiderivative size = 1809, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1809
default	Expression too large to display	35482

input
```
int((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(b/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^
3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+
b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2*(a^2*
c*f+a^2*d*e-b/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(g/h-a/
b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+
e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/
2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*E
llipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b
)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(2*a^2*d*f+2*a*c*f*b+2*a*b*d*e-b/h*(a*c*
f*h+a*d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b
)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/
d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a
/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h
+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-
c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/
d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h)
)^(1/2)))+(4*a*d*f*b+b^2*c*f+b^2*d*e-b/h*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*
e*h+3/2*b*d*f*g))*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(
-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/...
```

3.11.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.11.6 Sympy [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\,(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(c*f + d*e + 2*d*f*x)/(sqrt(c + d*x)*sqrt(e + f*
x)*sqrt(g + h*x)), x)
```

3.11.7 Maxima [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

3.11.8 Giac [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,(c\,f+d\,e+2\,d\,f\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/
2)*(c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(3/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/
2)*(c + d*x)^(1/2)), x)
```

3.12 $\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.12.1 Optimal result

Integrand size = 49, antiderivative size = 472

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}} - \frac{2\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{h\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} - \frac{2d(bg-ah)^{3/2}\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right),\frac{(de-cf)(}{(be-af)(}\right.}{\sqrt{be-af}\,h^2\sqrt{a+bx}\sqrt{c+dx}}$$

output
```
-2*d*(-a*h+b*g)^(3/2)*(f*x+e)*EllipticPi((-a*f+b*e)^(1/2)*(h*x+g)^(1/2)/(-
a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/(-a*f+b*e)/h,((-c*f+d*e)*(-a*h+b
*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*((-e*h+f*g)*(b*x+a)/(-a*h+b*g)/(f*x+e))^
(1/2)*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1/2)/h^2/(-a*f+b*e)^(1/2)/(
b*x+a)^(1/2)/(d*x+c)^(1/2)+2*b*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/h
/(b*x+a)^(1/2)-2*EllipticE((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)
/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h
+b*g)^(1/2)*(-e*h+f*g)^(1/2)*(d*x+c)^(1/2)*(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)
/(b*x+a))^(1/2)/h/((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1
/2)
```

3.12.2 Mathematica [A] (verified)

Time = 36.16 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{dh(e+fx)(g+hx)}{c+dx}-\frac{(fg-eh)\sqrt{\frac{(-de+cf)(dg-ch)(e+fx)(g+hx)}{(fg-eh)^2(c+dx)^2}}\left((de-cf)hE\left(\arcsin\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)|\frac{(bc-\cdots}{(de-\cdots}\right.\right.\cdots}{\cdots}\right.\cdots}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x]*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*
x]*Sqrt[g + h*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*Sqrt[c + d*x]*(-((d*h*(e + f*x)*(g + h*x))/(c + d*x)) -
((f*g - e*h)*Sqrt[((-(d*e) + c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g -
 e*h)^2*(c + d*x)^2)]*((d*e - c*f)*h*EllipticE[ArcSin[Sqrt[((-(d*e) + c*f)
*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e
- c*f)*(b*g - a*h))] + (-(d*e*h) + c*f*h)*EllipticF[ArcSin[Sqrt[((-(d*e) +
 c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/(
(d*e - c*f)*(b*g - a*h))] + f*(d*g - c*h)*EllipticPi[(d*(-(f*g) + e*h))/((
d*e - c*f)*h), ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*
x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))]))/((d*e - c
*f)*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])))/(h^2*Sqrt[e +
 f*x]*Sqrt[g + h*x])
```

3.12.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {2098, 183, 194, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{ 2098}$$

$$\frac{(be-af)(bg-ah)\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{d(bg-ah)\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}dx}{h}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

$$\Big\downarrow \text{183}$$

$$\frac{(be-af)(bg-ah)\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{2d(e+fx)(bg-ah)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{g+hx}}{\sqrt{e+fx}}}{h\sqrt{a+bx}\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

$$\Big\downarrow \text{194}$$

$$-\frac{2\sqrt{c+dx}(bg-ah)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\int \frac{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}}{\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{h\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}-\frac{2d(e+fx)(bg-ah)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{g+hx}}{\sqrt{e+fx}}}{h\sqrt{a+bx}\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2d(e+fx)(bg-ah)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{g+hx}}{\sqrt{e+fx}}}{h\sqrt{a+bx}\sqrt{c+dx}}-\frac{2\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{h\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

$$\Big\downarrow \text{412}$$

3.12. $\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{2d(e+fx)(bg-ah)^{3/2}\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{h^2\sqrt{a+bx}\sqrt{c+dx}\sqrt{be-af}}$$
$$\frac{2\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{h\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}+$$
$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

input
```
Int[(Sqrt[a + b*x]*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

output
```
(2*b*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(h*Sqrt[a + b*x]) - (2*Sqr
t[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/
((f*g - e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])
/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f
)*(b*g - a*h)))])/(h*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]
*Sqrt[g + h*x]) - (2*d*(b*g - a*h)^(3/2)*Sqrt[((f*g - e*h)*(a + b*x))/((b*
g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x))]
*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcSin[(Sqrt[b*e -
a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e - c*f)*(b*g -
a*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[b*e - a*f]*h^2*Sqrt[a + b*x]*Sqrt[
c + d*x])
```

3.12.3.1 Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2098
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*
B*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*Sqrt[a + b*x])), x] + (
-Simp[B*((b*g - a*h)/(2*f*h))   Int[Sqrt[e + f*x]/(Sqrt[a + b*x]*Sqrt[c + d
*x]*Sqrt[g + h*x]), x], x] + Simp[B*(b*e - a*f)*((b*g - a*h)/(2*d*f*h))   I
nt[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && EqQ[2*A*d*f - B*(d*e + c*f), 0]
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(426) = 852$.

Time = 5.17 (sec) , antiderivative size = 1560, normalized size of antiderivative = 3.31

method	result	size
elliptic	Expression too large to display	1560
default	Expression too large to display	13180

input
```
int((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*(a*c*f+a*d*e)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h
+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/
2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*
d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)
/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/
2))+2*(2*a*d*f+b*c*f+b*d*e)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/
d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a
/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/
b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/
h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(
c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/
b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+2*b*d*f*
((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))
^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)
*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/
h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),
((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((
-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f
)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f...
```

3.12.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.12.6 Sympy [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*(c*f + d*e + 2*d*f*x)/(sqrt(c + d*x)*sqrt(e + f*x)*
sqrt(g + h*x)), x)
```

3.12.7 Maxima [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)
*sqrt(h*x + g)), x)
```

3.12.8 Giac [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)
*sqrt(h*x + g)), x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\,x}\,(c\,f+d\,e+2\,d\,f\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/
2)*(c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(1/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/
2)*(c + d*x)^(1/2)), x)
```

3.13 $\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.13.1 Optimal result

Integrand size = 49, antiderivative size = 449

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(bde+bcf-2adf)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{4df\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-\cdots}{(bc-\cdots}\right.}{b\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
4*d*f*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b
*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(
-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^
(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b/h/(-a*d+b*c)^(1/2)/(
d*x+c)^(1/2)/(f*x+e)^(1/2)+2*(-2*a*d*f+b*c*f+b*d*e)*EllipticF((-a*h+b*g)^(
1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/
(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2
)*(h*x+g)^(1/2)/b/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+
b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 25.23 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.61

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\sqrt{a+bx}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-bde(be-af)h\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right),\frac{(-\cdots}{(\cdots}\right.\right.}{}$$

input
```
Integrate[(d*e + c*f + 2*d*f*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]
*Sqrt[g + h*x]),x]
```

output
```
(2*Sqrt[a + b*x]*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(-(
b*d*e*(b*e - a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*
(g + h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a
 + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]) +
2*a*d*f*(b*e - a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))
]*(g + h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*
(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] +
 b*c*f*(-(b*e) + a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x
))]*(g + h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h
)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]
 - 2*d*f*(b*g - a*h)*(f*g - e*h)*(a + b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))
/((f*g - e*h)*(a + b*x))]*Sqrt[((-(b*e) + a*f)*(b*g - a*h)*(e + f*x)*(g +
h*x))/((f*g - e*h)^2*(a + b*x)^2)]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a
*f)*h), ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]],
((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]))/(b*(b*e - a*f
)*h*(b*g - a*h)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*Sqrt[((-(b*e) +
a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])
```

3.13.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cf+de+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2101}$$

$$\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2df\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}$$

$$\Big\downarrow \text{ 183}$$

$$\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{4df(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{ 188}$$

$$\frac{2\sqrt{g+hx}(-2adf+bcf+bde)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{4df(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{4df(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{g+hx}(-2adf+bcf+bde)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{2\sqrt{g+hx}(-2adf+bcf+bde)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{4df(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{bh\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

input
```
Int[(d*e + c*f + 2*d*f*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]),x]
```

3.13. $\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(S
qrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(
b*g - a*h)))])/(b*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b
*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (4*d*f*Sqrt[-(d*g) + c*h
]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b
*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))
/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c
*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])
/(b*Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.13.3.1 Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

3.13.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(411) = 822$.

Time = 6.28 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.90

method	result
elliptic	$\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\left(\frac{2(cf+de)\left(\frac{g}{h}-\frac{a}{b}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}F\left(\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)(x}{\left(-\frac{g}{h}+\frac{a}{b}\right)(x}}\right.}{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}}\right.$
default	Expression too large to display

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*(c*f+d*e)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b
)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*(
(-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*
h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g
/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+
4*d*f*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((
-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)
/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+
g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),
((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((
-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/
d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))
```

3.13.5 Fricas [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.13.6 Sympy [F]

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{cf+de+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Integral((c*f + d*e + 2*d*f*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*
sqrt(g + h*x)), x)
```

3.13.7 Maxima [F]

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{2\,dfx+de+cf}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)
*sqrt(h*x + g)), x)
```

3.13.8 Giac [F]

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{2\,dfx+de+cf}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)
*sqrt(h*x + g)), x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{c\,f+d\,e+2\,d\,f\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)
*(c + d*x)^(1/2)),x)
```

output
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)
*(c + d*x)^(1/2)), x)
```

3.14 $\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.14.1 Optimal result

Integrand size = 49, antiderivative size = 625

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d(bde+bcf-2adf)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}$$
$$-\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$
$$-\frac{2(bde+bcf-2adf)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\,E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)\middle|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{2d(de-cf)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

output
```
2*d*(-2*a*d*f+b*c*f+b*d*e)*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d
+b*c)/(-a*f+b*e)/(-a*h+b*g)/(d*x+c)^(1/2)-2*b*(-2*a*d*f+b*c*f+b*d*e)*(d*x+
c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x
+a)^(1/2)-2*d*(-c*f+d*e)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*
g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2
))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/
(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h
+f*g)/(b*x+a))^(1/2)-2*(-2*a*d*f+b*c*f+b*d*e)*EllipticE((-c*h+d*g)^(1/2)*(
f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b
*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(
-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*
g)/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 25.78 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.55

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left((bde+bcf-\right.}{}$$

input
```
Integrate[(d*e + c*f + 2*d*f*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*
x]*Sqrt[g + h*x]),x]
```

output
```
(2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
f*x)^(3/2)*(g + h*x)^(3/2)*((b*d*e + b*c*f - 2*a*d*f)*(d*g - c*h)*Elliptic
E[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c
- a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))] - d*(d*e - c*f)*(b*g - a*h)
*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]
], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*
g - e*h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e +
f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

3.14.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 595, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cf+de+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2102}$$

$$\frac{\int \frac{-df(de+cf)ha^2-b\left(e(fg-eh)d^2+cf^2gd-c^2f^2h\right)a+2bdf(bde+bcf-2adf)hx^2+2b^2cdefg+(bde+bcf-2adf)(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{2105}$$

$$\frac{\frac{\int -\frac{2bd^2f(be-af)(de-cf)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+(de-cf)(dg-ch)(-2adf+bcf+bde)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\cdots}{}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 27

$$\frac{(de-cf)(dg-ch)(-2adf+bcf+bde)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx-d(be-af)(bg-ah)(de-cf)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\cdots}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 188

$$\frac{(de-cf)(dg-ch)(-2adf+bcf+bde)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{2d\sqrt{g+hx}(be-af)(bg-ah)(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \cdots}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+h\cdots}{(a+b\cdots}}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 194

$$\frac{-\frac{2\sqrt{a+bx}(dg-ch)(-2adf+bcf+bde)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}-\frac{2d\sqrt{g+hx}(be-af)(bg-ah)(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \cdots}{\sqrt{c+dx}(fg-eh)\sqrt{-\cdots}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 321

$$\frac{-\frac{2\sqrt{a+bx}(dg-ch)(-2adf+bcf+bde)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}-\frac{2d\sqrt{g+hx}(be-af)\sqrt{bg-ah}(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{E}\cdots}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{\cdots}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 327

3.14. $\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{-\frac{2d\sqrt{g+hx}(be-af)\sqrt{bg-ah}(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}} - \frac{2\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}(-2adf \cdots}{\cdots}}{(bc-ad)(be-af)(bg \cdots}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

input
```
Int[(d*e + c*f + 2*d*f*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

output
```
(-2*b*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*d*(b*d*e + b*c*
f - 2*a*d*f)*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2
*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*S
qrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(
Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a
*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[((d*e - c*f)*(a + b*x))
/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - (2*d*(b*e - a*f)*(d*e - c*f)*Sq
rt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g
 + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*
Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/
(Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)
*(a + b*x)))]))/((b*c - a*d)*(b*e - a*f)*(b*g - a*h))
```

3.14.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.14. $\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.14.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. $2(571) = 1142$.

Time = 7.75 (sec) , antiderivative size = 2298, normalized size of antiderivative = 3.68

method	result	size
elliptic	Expression too large to display	2298
default	Expression too large to display	21256

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(-2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2
+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h
-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f
-b*d*e)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*
x+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(2/b*d*f-1/b*(a^2*d*f*h-a*b*c*f*h-
a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(2*a*d*f-b*c*f-b*d*e)/(
a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^
2*d*e*g-b^3*c*e*g)+(b*c*e*h+b*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*
d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-
b*c*f-b*d*e))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c
/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-
g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+
e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2
),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*((a*d*f*h-b*c*f*h-b
*d*e*h-b*d*f*g)*(2*a*d*f-b*c*f-b*d*e)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a
^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)+(2*b*c*f*h+2*b*d
*e*h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h
+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f-b*d*e))*(g/h-a/b)*((-g/
h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/...
```

3.14.5 Fricas [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx+de+cf}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
integral((2*d*f*x + d*e + c*f)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)/(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c +
 2*a*b*d)*f)*h)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b
*d)*e + (2*a*b*c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a
^2*d)*f)*g + (a^2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*
f + (2*a*b*c + a^2*d)*e)*g)*x), x)
```

3.14.6 Sympy [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{cf+de+2dfx}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Integral((c*f + d*e + 2*d*f*x)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*
x)*sqrt(g + h*x)), x)
```

3.14.7 Maxima [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx+de+cf}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

3.14.8 Giac [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx+de+cf}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{c\,f+d\,e+2\,d\,f\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)
*(c + d*x)^(1/2)),x)
```

output
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)
*(c + d*x)^(1/2)), x)
```

3.15 $\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.15.1 Optimal result

Integrand size = 49, antiderivative size = 1090

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{4d(3a^3d^2f^2h-a^2bdf(dfg+4deh+4cfh)-b^3(d^2e^2g-cde(f\ldots}{\ldots}$$

$$-\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{4b(3a^3d^2f^2h-a^2bdf(dfg+4deh+4cfh)-b^3(d^2e^2g-cde(fg-eh)+c^2f(fg+eh))+ab^2(2c^2f^2h+d^2\ldots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$-\frac{4\sqrt{dg-ch}\sqrt{fg-eh}(3a^3d^2f^2h-a^2bdf(dfg+4deh+4cfh)-b^3(d^2e^2g-cde(fg-eh)+c^2f(fg+eh))\ldots}{3(bc-ad)^2(be-af)^2(\ldots}$$

$$+\frac{2(de-cf)\left(3a^2d^2fh-abd(dfg+3deh+2cfh)+b^2(2d^2eg-cdfg+cdeh+c^2fh)\right)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+h\ldots}}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-a\ldots}{(fg-e\ldots}}}$$

output
```
4/3*d*(3*a^3*d^2*f^2*h-a^2*b*d*f*(4*c*f*h+4*d*e*h+d*f*g)-b^3*(d^2*e^2*g-c*
d*e*(-e*h+f*g)+c^2*f*(e*h+f*g))+a*b^2*(2*c^2*f^2*h+d^2*e*(2*e*h+f*g)+c*d*f
*(3*e*h+f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*
f+b*e)^2/(-a*h+b*g)^2/(d*x+c)^(1/2)-2/3*b*(-2*a*d*f+b*c*f+b*d*e)*(d*x+c)^(
1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^
(3/2)-4/3*b*(3*a^3*d^2*f^2*h-a^2*b*d*f*(4*c*f*h+4*d*e*h+d*f*g)-b^3*(d^2*e^
2*g-c*d*e*(-e*h+f*g)+c^2*f*(e*h+f*g))+a*b^2*(2*c^2*f^2*h+d^2*e*(2*e*h+f*g)
+c*d*f*(3*e*h+f*g)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^
2/(-a*f+b*e)^2/(-a*h+b*g)^2/(b*x+a)^(1/2)+2/3*(-c*f+d*e)*(3*a^2*d^2*f*h-a*
b*d*(2*c*f*h+3*d*e*h+d*f*g)+b^2*(c^2*f*h+c*d*e*h-c*d*f*g+2*d^2*e*g))*Ellip
ticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d
+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d
*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)^(3/2)/
(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1
/2)-4/3*(3*a^3*d^2*f^2*h-a^2*b*d*f*(4*c*f*h+4*d*e*h+d*f*g)-b^3*(d^2*e^2*g-
c*d*e*(-e*h+f*g)+c^2*f*(e*h+f*g))+a*b^2*(2*c^2*f^2*h+d^2*e*(2*e*h+f*g)+c*d
*f*(3*e*h+f*g)))*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)
/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+
d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/
(d*x+c))^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/((-c*f+d*e)*(b*x+...
```

3.15.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10790 vs. 2(1090) = 2180.

Time = 38.04 (sec) , antiderivative size = 10790, normalized size of antiderivative = 9.90

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*e + c*f + 2*d*f*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*
x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

3.15.3 Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 1077, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2102, 2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cf+de+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int \frac{2bdf(3bceg-a(deg+cfg+ceh))-(de+cf)\left(3dfha^2-3b(dfg+deh+cfh)a+2b^2(deg+cfg+ceh)\right)+(bde+bcf-2adf)(3adfh-b(dfg+deh+cfh))x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2102}$$

$$\int \frac{4bdfh\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\left(f(fg+eh)c^2-de(fg-eh)c+d^2e^2g\right)\right)x^2+2(adfh+b(dfg+deh+cfh))}{}$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 2105}$$

$$2(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\left(f(fg+eh)c^2\right.\right.$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$(be-af)(bg-ah)(de-cf)\left(3a^2d^2fh-abd(2cfh+3deh+dfg)+b^2\left(c^2fh+cdeh-cdfg+2d^2eg\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+2(de-cf)(dg-ch)(3a$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 188}$$

$$2(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\left(f(fg+eh)c^2\right.\right.$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\big\downarrow$ 194

$$-\frac{4(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\left(f(\right.\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\big\downarrow$ 321

$$-\frac{4(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\left(f(\right.\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\big\downarrow$ 327

$$-\frac{4\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3e\right.\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

input
```
Int[(d*e + c*f + 2*d*f*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

3.15. $\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(-2*b*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
/(3*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) + ((-4*b*(3*a^3*d
^2*f^2*h - a^2*b*d*f*(d*f*g + 4*d*e*h + 4*c*f*h) - b^3*(d^2*e^2*g - c*d*e*
(f*g - e*h) + c^2*f*(f*g + e*h)) + a*b^2*(2*c^2*f^2*h + d^2*e*(f*g + 2*e*h
) + c*d*f*(f*g + 3*e*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c
 - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((4*d*(3*a^3*d^2*f^2*h -
a^2*b*d*f*(d*f*g + 4*d*e*h + 4*c*f*h) - b^3*(d^2*e^2*g - c*d*e*(f*g - e*h)
 + c^2*f*(f*g + e*h)) + a*b^2*(2*c^2*f^2*h + d^2*e*(f*g + 2*e*h) + c*d*f*(
f*g + 3*e*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] -
(4*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(3*a^3*d^2*f^2*h - a^2*b*d*f*(d*f*g + 4
*d*e*h + 4*c*f*h) - b^3*(d^2*e^2*g - c*d*e*(f*g - e*h) + c^2*f*(f*g + e*h)
) + a*b^2*(2*c^2*f^2*h + d^2*e*(f*g + 2*e*h) + c*d*f*(f*g + 3*e*h)))*Sqrt[
a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*Elliptic
E[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])],
 ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[((d*e - c*f)*
(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) + (2*(b*e - a*f)*(d*e -
 c*f)*Sqrt[b*g - a*h]*(3*a^2*d^2*f*h - a*b*d*(d*f*g + 3*d*e*h + 2*c*f*h) +
 b^2*(2*d^2*e*g - c*d*f*g + c*d*e*h + c^2*f*h))*Sqrt[((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h
]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g ...
```

3.15.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.15. $\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3570 vs. $2(1018) = 2036$.

Time = 9.42 (sec) , antiderivative size = 3571, normalized size of antiderivative = 3.28

method	result	size
elliptic	Expression too large to display	3571
default	Expression too large to display	87910

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(-2/3/b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f
*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f-b*d*e)*(b
*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d
*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g
*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2-4/3*(b*d*f*h*x^3+b*c*f*h*x
^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f
*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g
-b^3*c*e*g)^2*(3*a^3*d^2*f^2*h-4*a^2*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d
^2*f^2*g+2*a*b^2*c^2*f^2*h+3*a*b^2*c*d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e
^2*h+a*b^2*d^2*e*f*g-b^3*c^2*e*f*h-b^3*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f
*g-b^3*d^2*e^2*g)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^
2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(1/3/b*(6*a^2*d^2*f^2*h-
5*a*b*c*d*f^2*h-5*a*b*d^2*e*f*h-2*a*b*d^2*f^2*g+b^2*c^2*f^2*h+2*b^2*c*d*e*
f*h+b^2*c*d*f^2*g+b^2*d^2*e^2*h+b^2*d^2*e*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*
b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-2/3/b*(
a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(3*
a^3*d^2*f^2*h-4*a^2*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d^2*f^2*g+2*a*b^2*
c^2*f^2*h+3*a*b^2*c*d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e^2*h+a*b^2*d^2*e*
f*g-b^3*c^2*e*f*h-b^3*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f*g-b^3*d^2*e^2...
```

3.15.5 Fricas [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
integral((2*d*f*x + d*e + c*f)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c +
 3*a*b^2*d)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3
*a*b^2*d)*e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e +
3*(a*b^2*c + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d
)*f)*h)*x^3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*
c*f + (3*a^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c +
 a^3*d)*e)*g)*x), x)
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \text{Timed out}$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Timed out
```

3.15.7 Maxima [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

3.15.8 Giac [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \int \frac{2\, dfx + de + cf}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\, dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \text{Hanged}$$

input
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)
*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.16 $\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.16.1 Optimal result

Integrand size = 58, antiderivative size = 721

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2b^2(5bBdfh+2C(adfh-2b(dfg+deh+cfh)))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$

$$+ \frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$- \frac{2b\sqrt{-de+cf}(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(dfg+deh+cfh))+b^2(10Bdfh(dfg+deh+cfh}{15d^3f^{5/2}$$

$$- \frac{2\sqrt{-de+cf}(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Cg+Bh)+5ab^2dfh(6Bdfgh-C(ch(fg-eh)+dg(2fg+}{}$$

output
```
2/15*b^2*(5*b*B*d*f*h+2*C*(a*d*f*h-2*b*(c*f*h+d*e*h+d*f*g)))*(d*x+c)^(1/2)
*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+2/5*b^2*C*(b*x+a)*(d*x+c)^(1/2)*(
f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-2/15*b*(15*a^2*C*d^2*f^2*h^2-10*a*b*d*f*h
*(3*B*d*f*h-C*(c*f*h+d*e*h+d*f*g))+b^2*(10*B*d*f*h*(c*f*h+d*e*h+d*f*g)-C*(
8*c^2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f^2*g^2))))*E
llipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))
^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^3/f^(
5/2)/h^3/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2/15*(15*a^3*C*d^2*f^2
*h^3-15*a^2*b*d^2*f^2*h^2*(B*h+C*g)+5*a*b^2*d*f*h*(6*B*d*f*g*h-C*(c*h*(-e*
h+f*g)+d*g*(e*h+2*f*g)))-b^3*(5*B*d*f*h*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g))-C
*(4*c^2*f*h^2*(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2
*h^2+3*e*f*g*h+8*f^2*g^2))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/
2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e
))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d^3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g
)^(1/2)
```

3.16.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.44 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(bd^2\sqrt{-c+\frac{de}{f}}(15a^2Cd^2f^2h^2+10abdfh(-3Bdfh+C(dfg+deh+cfh))-b^2(-10Bdfh(dfg+deh\right.}{}$$

input
```
Integrate[((a + b*x)*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*(b*d^2*Sqrt[-c + (d*e)/f]*(15*a^2*C*d^2*f^2*h^2 + 10*a*b*d*f*h*(-3*B*d
*f*h + C*(d*f*g + d*e*h + c*f*h)) - b^2*(-10*B*d*f*h*(d*f*g + d*e*h + c*f*
h) + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h
 + 8*e^2*h^2))))*(e + f*x)*(g + h*x) + b^2*d^2*Sqrt[-c + (d*e)/f]*f*h*(c +
 d*x)*(e + f*x)*(g + h*x)*(-5*b*B*d*f*h - 5*a*C*d*f*h + b*C*(4*c*f*h + d*(
4*f*g + 4*e*h - 3*f*h*x))) + I*b*(d*e - c*f)*h*(15*a^2*C*d^2*f^2*h^2 + 10*
a*b*d*f*h*(-3*B*d*f*h + C*(d*f*g + d*e*h + c*f*h)) - b^2*(-10*B*d*f*h*(d*f
*g + d*e*h + c*f*h) + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^
2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c
 + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c +
(d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*d*h*(15*a^3*
C*d^2*f^3*h^2 - 15*a^2*b*d^2*f^2*(C*e + B*f)*h^2 - 5*a*b^2*d*f*h*(-6*B*d*e
*f*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) + b^3*(-5*B*d*f*h*(c*f*
(-(f*g) + e*h) + d*e*(f*g + 2*e*h)) + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*
f*(-4*f^2*g^2 + e*f*g*h + 3*e^2*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^
2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h
*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]],
 (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(15*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqr
t[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

3.16.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2004, 2100, 2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(a^2(-C)+abB+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{(a+bx)^2\left(\frac{abB-a^2C}{a}+bCx\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2100}$$

$$\int \frac{5(bB-aC)dfha^2+b^2(5bBdfh+2aCdfh-4bC(dfg+deh+cfh))x^2-b^2C(2bceg+a(deg+cfg+ceh))-b\big(5Cdfha^2-2b(5Bdfh-C(dfg+deh+cfh))a}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}$$

$$5dfh$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \; 2118$$

$$2\int \frac{d\Big(-15Cd^2f^2h^2a^3+15bBd^2f^2h^2a^2-5b^2Cdfh(deg+cfg+ceh)a-b^3\Big(5Bdfh(deg+cfg+ceh)-C\Big(4fh(fg+eh)c^2+2d\Big(2f^2g^2+3efhg+2e^2h^2\Big)c+4d^2eg(fg+eh)\Big)\Big)}{2\sqrt{c+dx}\sqrt{e+fx}}$$

$$3d^2fh$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \; 27$$

$$\int \frac{-15Cd^2f^2h^2a^3+15bBd^2f^2h^2a^2-5b^2Cdfh(deg+cfg+ceh)a-b^3\Big(5Bdfh(deg+cfg+ceh)-C\Big(4fh(fg+eh)c^2+2d\Big(2f^2g^2+3efhg+2e^2h^2\Big)c+4d^2eg(fg+eh)\Big)\Big)-b\Big(}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+}}$$

$$3dfh$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \; 176$$

$$-\frac{b\Big(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(cfh+deh+dfg))+b^2\Big(10Bdfh(cfh+deh+dfg)-C\Big(8c^2f^2h^2+7cdfh(eh+fg)+d^2\Big(8e^2h^2+7efgh+8f^2g^2\Big)\Big)\Big)\Big)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+f}}}{h}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \; 124$$

$$-\frac{b\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\Big(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(cfh+deh+dfg))+b^2\Big(10Bdfh(cfh+deh+dfg)-C\Big(8c^2f^2h^2+7cdfh(eh+fg)+d^2\Big(8e^2h^2+7efgh+8f^2g^2\Big)}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \; 123$$

3.16. $\displaystyle\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{\left(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Bh+Cg)+5ab^2dfh(6Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))-\left(b^3\left(5Bdfh(ch(fg-eh)+dg(eh+2fg))-C\left(4c^2fh^2(fg-eh)+cdh\left(-4e\right.\right.\right.\right.}{h}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 131

$$-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\left(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Bh+Cg)+5ab^2dfh(6Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))-\left(b^3\left(5Bdfh(ch(fg-eh)+dg(eh+2fg))-C\left(4c^2fh^2(fg-eh\right.\right.\right.\right.}{h\sqrt{e+fx}}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 131

$$-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Bh+Cg)+5ab^2dfh(6Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))-\left(b^3\left(5Bdfh(ch(fg-eh)+dg(eh+2fg))-C\left(4c^2\right.\right.\right.\right.}{h\sqrt{e+fx}\sqrt{g+hx}}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 130

$$-\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(cfh+deh+dfg))+b^2\left(10Bdfh(cfh+deh+dfg)-C\left(8c^2f^2h^2\right.\right.\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

input
```
Int[((a + b*x)*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]*Sqrt[
e + f*x]*Sqrt[g + h*x]),x]
```

3.16. $\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*b^2*C*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) +
((2*b^2*(5*b*B*d*f*h + 2*a*C*d*f*h - 4*b*C*(d*f*g + d*e*h + c*f*h))*Sqrt[c
 + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((-2*b*Sqrt[-(d*e) + c*f]
*(15*a^2*C*d^2*f^2*h^2 - 10*a*b*d*f*h*(3*B*d*f*h - C*(d*f*g + d*e*h + c*f*
h)) + b^2*(10*B*d*f*h*(d*f*g + d*e*h + c*f*h) - C*(8*c^2*f^2*h^2 + 7*c*d*f
*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*Sqrt[(d*(e + f
*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/S
qrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e
+ f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]*(15*a^3*C*
d^2*f^2*h^3 - 15*a^2*b*d^2*f^2*h^2*(C*g + B*h) + 5*a*b^2*d*f*h*(6*B*d*f*g*
h - c*C*h*(f*g - e*h) - C*d*g*(2*f*g + e*h)) - b^3*(5*B*d*f*h*(c*h*(f*g -
e*h) + d*g*(2*f*g + e*h)) - C*(4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2
+ e*f*g*h - 4*e^2*h^2) + d^2*g*(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt
[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcS
in[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]))/(3*d*f*h))/(5*d*f*h)
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

3.16. $\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.16.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.22

method	result
elliptic	$\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2C\,b^3x\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(B\,b^3+C\,b^2a-\frac{2C\,b^3(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{dfh}}{\ldots}\right.$
default	Expression too large to display

input
```
int((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h
*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2/5*C*b^3/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*
x+d*e*g*x+c*e*g)^(1/2)+2/3*(B*b^3+C*b^2*a-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h
+2*d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2*(a^2*b*B-C*a^3-2/5*C*b^3/d/f/h*c*e*g-2/3*(B*b^3+C*b
^2*a-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g
+1/2*d*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2
)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*
h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/
h+e/f)/(-g/h+c/d))^(1/2))+2*(2*a*b^2*B-C*a^2*b-2/5*C*b^3/d/f/h*(3/2*c*e*h+
3/2*c*f*g+3/2*d*e*g)-2/3*(B*b^3+C*b^2*a-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h+2
*d*f*g))/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((
x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d
*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*Ellipt
icE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF
(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

3.16.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1267, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*(3*C*b^3*d^3*f^3*h^3*x - 4*C*b^3*d^3*f^3*g*h^2 - (4*C*b^3*d^3*e*f^
2 + (4*C*b^3*c*d^2 - 5*(C*a*b^2 + B*b^3)*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt
(f*x + e)*sqrt(h*x + g) - (8*C*b^3*d^3*f^3*g^3 + (3*C*b^3*d^3*e*f^2 + (3*C
*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*f^3)*g^2*h + (3*C*b^3*d^3*e^2*f + (
3*C*b^3*c*d^2 - 5*(C*a*b^2 + B*b^3)*d^3)*e*f^2 + (3*C*b^3*c^2*d - 5*(C*a*b
^2 + B*b^3)*c*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*f^3)*g*h^2 + (8*C*b^3*d^
3*e^3 + (3*C*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*e^2*f + (3*C*b^3*c^2*d
- 5*(C*a*b^2 + B*b^3)*c*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*e*f^2 + (8*C*b
^3*c^3 - 10*(C*a*b^2 + B*b^3)*c^2*d - 15*(C*a^2*b - 2*B*a*b^2)*c*d^2 + 45*
(C*a^3 - B*a^2*b)*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*
f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^
2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^
3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f -
3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d
*e + c*f)*h)/(d*f*h)) - 3*(8*C*b^3*d^3*f^3*g^2*h + (7*C*b^3*d^3*e*f^2 + (7
*C*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*f^3)*g*h^2 + (8*C*b^3*d^3*e^2*f +
 (7*C*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*e*f^2 + (8*C*b^3*c^2*d - 10*(C
*a*b^2 + B*b^3)*c*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*f^3)*h^3)*sqrt(d*f*h
)*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 -
c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*...
```

3.16.6 Sympy [F]

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(a+bx)^2\,(Bb-Ca+Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)*(C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2)/(f*x+
e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)**2*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqr
t(g + h*x)), x)
```

3.16.7 Maxima [F]

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*(b*x + a)/(sqrt(d*x + c)*s
qrt(f*x + e)*sqrt(h*x + g)), x)
```

3.16.8 Giac [F]

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*(b*x + a)/(sqrt(d*x + c)*s
qrt(f*x + e)*sqrt(h*x + g)), x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Hanged}$$

input
```
int(((a + b*x)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/2)*(g
+ h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.17 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.17.1 Optimal result

Integrand size = 53, antiderivative size = 410

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$
$$+\frac{2b^2\sqrt{-de+cf}(3Bdfh-2C(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{2\sqrt{-de+cf}(3abBdfh^2-3a^2Cdfh^2-b^2(3Bdfgh-C(ch(fg-eh)+dg(2fg+eh))))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+ \dots}{dg- \dots}}}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2/3*b^2*C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+2/3*b^2*(3*B*d*f
*h-2*C*(c*f*h+d*e*h+d*f*g))*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2
),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e)
)^(1/2)*(h*x+g)^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))
^(1/2)+2/3*(3*a*b*B*d*f*h^2-3*a^2*C*d*f*h^2-b^2*(3*B*d*f*g*h-C*(c*h*(-e*h+
f*g)+d*g*(e*h+2*f*g))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((
-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1
/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(h*x+g)^(1/
2)
```

3.17.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.00 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.08

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\sqrt{c+dx}\left(2b^2Cd^2fh(e+fx)(g+hx) + \frac{2b^2d^2(3Bdfh-2C(dfg+deh+cfh))(e+fx)(g+hx)}{c+dx} + 2ib^2\sqrt{-c+\frac{de}{f}}fh(3Bdfh\right.}{}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*
x]*Sqrt[g + h*x]),x]
```

output
```
(Sqrt[c + d*x]*(2*b^2*C*d^2*f*h*(e + f*x)*(g + h*x) + (2*b^2*d^2*(3*B*d*f*
h - 2*C*(d*f*g + d*e*h + c*f*h))*(e + f*x)*(g + h*x))/(c + d*x) + (2*I)*b^
2*Sqrt[-c + (d*e)/f]*f*h*(3*B*d*f*h - 2*C*(d*f*g + d*e*h + c*f*h))*Sqrt[c
+ d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]
*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d
*e*h - c*f*h)] + ((2*I)*d*h*(3*a*b*B*d*f^2*h - 3*a^2*C*d*f^2*h + b^2*(-3*B
*d*e*f*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)))*Sqrt[c + d*x]*Sqrt
[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[
I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*
h)])/Sqrt[-c + (d*e)/f]))/(3*d^3*f^2*h^2*Sqrt[e + f*x]*Sqrt[g + h*x])
```

3.17.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.151$, Rules used = {2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C) + abB + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \; 2118$$

$$\frac{2\int \frac{d\left(-3Cdfha^2+3bBdfha-b^2C(deg+cfg+ceh)+b^2(3Bdfh-2C(dfg+deh+cfh))x\right)}{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3d^2fh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 27

$$\frac{\int \frac{-3Cdfha^2+3bBdfha-b^2C(deg+cfg+ceh)+b^2(3Bdfh-2C(dfg+deh+cfh))x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 176

$$\frac{\frac{\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}+\frac{b^2(3Bdfh-2C(cfh+deh+dfg))\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e\cdots}}}{h}}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 124

$$\frac{\frac{\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}+\frac{b^2\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}(3Bdfh-2C(cfh+deh+\cdots}{h\sqrt{e+fx}\sqrt{\frac{d(g\cdots}{dg\cdots}}}}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 123

$$\frac{\frac{\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}+\frac{2b^2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(3Bdfh-2C(\cdots}{d\sqrt{f}h\sqrt{e\cdots}}}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 131

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}+2b^2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d\cdots}{\cdots}}}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

3.17. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$\downarrow$ 131

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2b^2}{}}{3dfh}$$

$$\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 130

$$\frac{\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}}{3dfh}$$

$$\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

output
```
(2*b^2*C*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((2*b^2*Sq
rt[-(d*e) + c*f]*(3*B*d*f*h - 2*C*(d*f*g + d*e*h + c*f*h))*Sqrt[(d*(e + f*
x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e +
 f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a*b*B*d*
f*h^2 - 3*a^2*C*d*f*h^2 - b^2*(3*B*d*f*g*h - c*C*h*(f*g - e*h) - C*d*g*(2*
f*g + e*h)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h
)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c
*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]))/(3*d*f
*h)
```

3.17. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.17.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.55

method	result
elliptic	$\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2C\,b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(abB-C\,a^2-\frac{2C\,b^2\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{g}{h}-\right.}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x}}\right.$
default	Expression too large to display

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2/3*C*b^2/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2*(a*b*B-C*a^2-2/3*C*b^2/d/f/h*(1/2*c*e*h+1/2*c*f*g+1
/2*d*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*
((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*
x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+
e/f)/(-g/h+c/d))^(1/2))+2*(B*b^2-2/3*C*b^2/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h
-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+
e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g
*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f
)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(
-g/h+c/d))^(1/2))))
```

3.17.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.10

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx$$
$$= \frac{2\left(3\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}Cb^2d^2f^2h^2 + (2\,Cb^2d^2f^2g^2 + (Cb^2d^2ef + (Cb^2cd - 3\,Bb^2d^2)f^2)gh + (2\,Cb\right.}{}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*C*b^2*d^2*f^2*h^2 + (2*C*
b^2*d^2*f^2*g^2 + (C*b^2*d^2*e*f + (C*b^2*c*d - 3*B*b^2*d^2)*f^2)*g*h + (2
*C*b^2*d^2*e^2 + (C*b^2*c*d - 3*B*b^2*d^2)*e*f + (2*C*b^2*c^2 - 3*B*b^2*c*
d - 9*(C*a^2 - B*a*b)*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(
d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)
/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3
*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*
f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g
+ (d*e + c*f)*h)/(d*f*h)) + 3*(2*C*b^2*d^2*f^2*g*h + (2*C*b^2*d^2*e*f + (2
*C*b^2*c*d - 3*B*b^2*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f
^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2
*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3
*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3
*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2
*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d
^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d
^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f -
 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (
d*e + c*f)*h)/(d*f*h))))/(d^3*f^3*h^3)
```

3.17.6 Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \int \frac{(a + bx)\,(Bb - Ca + Cbx)}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2)/(f*x+e)**(1/2
)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
 + h*x)), x)
```

3.17.7 Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

3.17.8 Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \text{Hanged}$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.18 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.18.1 Optimal result

Integrand size = 60, antiderivative size = 291

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2bC\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(bCg-bBh+aCh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*b*C*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*
h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/
d/h/f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(-B*b*h+C*a*h+C*b
*g)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h
+d*g))^(1/2)/d/h/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.18.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.12

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\left(bCd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx) + ibC(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right)\right.\right.}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{c}}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(b*C*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x) + I*b*C*(d*e - c*f)*h*(
c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
+ d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c
*f*h)/(d*e*h - c*f*h)] - I*d*(b*C*e - b*B*f + a*C*f)*h*(c + d*x)^(3/2)*Sqr
t[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF
[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f
*h)]))/(d^2*Sqrt[-c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x])
```

3.18.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.117$, Rules used = {2004, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C) + abB + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$\Big\downarrow \text{2004}$$
$$\int \frac{\frac{abB-a^2C}{a} + bCx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$
$$\Big\downarrow \text{176}$$

$$\frac{bC\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}-\frac{(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{124}$$

$$\frac{bC\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{123}$$

$$\frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{131}$$

$$\frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}$$

$$\Big\downarrow \text{131}$$

$$\frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{130}$$

$$\frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(aCh-bBh+bCg)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]),x]
```

3.18. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*b*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*El
lipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h
)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c
*h)]) - (2*Sqrt[-(d*e) + c*f]*(b*C*g - b*B*h + a*C*h)*Sqrt[(d*(e + f*x))/(
d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt
[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[
f]*h*Sqrt[e + f*x]*Sqrt[g + h*x])
```

3.18.3.1 Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

3.18.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2(Bb-Ca)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfhx^3+cfhx^2+dehx^2+dfgx^2+cehx+cfgx+degx+ceg}}+\frac{2Cb\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\cdots}}{\sqrt{dfhx^3+\cdots}}\cdots\right)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}$
default	$-\frac{2\left(BF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)bdeh^2-BF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)bdfgh-CF\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)adeh^2+CF\left(\cdots\right)\cdots\right)}{\cdots}$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h
*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2*(B*b-C*a)*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2
)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*
h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/
h+e/f)/(-g/h+c/d))^(1/2))+2*C*b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/
d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h
*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(
((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x
+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

3.18.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.34

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(3\sqrt{dfh}Cbdfh\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-(d^2ef+cdf^2)gh+(d^2e^2-cdef+c^2f^2)h^2\right)}{3\,d^2f^2h^2}, -\frac{4\left(2\,d^3f^3g^3-3\,(d^3ef^2+cd^2f^3)g^2h-3\right.}{}\right.\right.}{}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(d*f*h)*C*b*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f
+ c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(
2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e
*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c
^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*
f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27
*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2
*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2
*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f
*h))) + (C*b*d*f*g + (C*b*d*e + (C*b*c + 3*(C*a - B*b)*d)*f)*h)*sqrt(d*f*h
)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^
2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e
*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2
+ (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^
3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)))/(d^2*f^2*h^2)
```

3.18.6 Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bb - Ca + Cbx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)/(d*x+c)**(1/2)/(f*x+
e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)),
x)
```

3.18.7 Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)*sqrt(d*x + c)*s
qrt(f*x + e)*sqrt(h*x + g)), x)
```

3.18.8 Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)*sqrt(d*x + c)*s
qrt(f*x + e)*sqrt(h*x + g)), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Hanged}$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.19 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.19.1 Optimal result

Integrand size = 60, antiderivative size = 309

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2(bB-2aC)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*C*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h
+d*g))^(1/2)/d/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(B*b-2*C*a)*EllipticP
i(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+
d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(
d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.19.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.06 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.81

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2i\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(-\left((bcC - bBd + aCd)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right), \frac{dfg-cfh}{deh-cfh}\right)\right) + (-bB + 2aC)\right.}{(-bc+ad)\sqrt{-c+\frac{de}{f}}f\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{g+hx}}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^2*Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
((2*I)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*(-((b*c*C - b*B*d +
 a*C*d)*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*
f*h)/(d*e*h - c*f*h)]) + (-(b*B) + 2*a*C)*d*EllipticPi[-((b*c*f - a*d*f)/(
b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f
*h)/(d*e*h - c*f*h)]))/((-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*f*Sqrt[(d*(e + f
*x))/(f*(c + d*x))]*Sqrt[g + h*x])
```

3.19.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2004, 2110, 27, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C) + abB + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 2004$$

$$\int \frac{\frac{abB-a^2C}{a} + bCx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \ 2110$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\int \frac{C}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \textbf{27}$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+C\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \textbf{131}$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{C\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{\sqrt{e+fx}}$$

$$\Big\downarrow \textbf{131}$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{C\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \textbf{130}$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \textbf{187}$$

$$\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-2(bB-2aC)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}$$

$$\Big\downarrow \textbf{413}$$

$$\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bB-2aC)\sqrt{\frac{f(c+dx)}{de-cf}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}$$

$$\Big\downarrow \textbf{413}$$

3.19. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bB-2aC)\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int\frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bB-2aC)\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\operatorname{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[
e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))
/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]]
, ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]
) - (2*(b*B - 2*a*C)*Sqrt[-(d*e) + c*f]*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)
]*Sqrt[1 + (h*(c + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c -
 a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f
)*h)/(f*(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x
))/d]*Sqrt[g - (c*h)/d + (h*(c + d*x))/d])
```

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

3.19. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.19.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2C\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2(Bb-2Ca)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{-}}{b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+c}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$-\frac{2\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}\,\sqrt{-\frac{(hx+g)f}{eh-fg}}\,\sqrt{\frac{(dx+c)h}{ch-dg}}\,\sqrt{\frac{(fx+e)h}{eh-fg}}\left(B\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(eh-fg)b}{f(ah-gb)},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)be\,h^2-B\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\right.\right.}{}$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/
(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2*C*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/
f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(
-g/h+c/d))^(1/2))+2*(B*b-2*C*a)/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+
c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e
*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*EllipticP
i(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^
(1/2)))
```

3.19.5 Fricas [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**2/(d*x+c)**(1/2)/(f
*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Timed out
```

3.19.7 Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^2*sqrt(d*x + c)
*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.19.8 Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^2*sqrt(d*x + c)
*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Hanged}$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^2*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.20 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.20.1 Optimal result

Integrand size = 60, antiderivative size = 680

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{b(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{\sqrt{-de+cf}(4a^3Cdfh+2ab^2B(dfg+deh+cfh)-b^3(Bdeg-c(2Ceg-Bfg-Beh))-a^2b(3Bdfh+ \ldots}{(bc-ad)^2\sqrt{f}(be-af)(bg- \ldots}$$

output
```
-b^2*(B*b-2*C*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*
f+b*e)/(-a*h+b*g)/(b*x+a)+b*(B*b-2*C*a)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c
*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(c*f-d*e)^(1/2)*(
d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)
/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-(4*a^3*C*d*f*h+2*a*b^2*B*(c*f*
h+d*e*h+d*f*g)-b^3*(B*d*e*g-c*(-B*e*h-B*f*g+2*C*e*g))-a^2*b*(3*B*d*f*h+2*C
*(c*f*h+d*e*h+d*f*g)))*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b
*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2
)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)^2/(
-a*f+b*e)/(-a*h+b*g)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-(B*b-2*C*a)*Ellip
ticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/
2))*f^(1/2)*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+
d*g))^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.20.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.66 (sec) , antiderivative size = 3419, normalized size of antiderivative = 5.03

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^3*Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
-((b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*
d)*(b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((c + d*x)^(3/2)*(b^3*B*c*Sqrt[-c
 + (d*e)/f]*f*h - 2*a*b^2*c*C*Sqrt[-c + (d*e)/f]*f*h - a*b^2*B*d*Sqrt[-c +
 (d*e)/f]*f*h + 2*a^2*b*C*d*Sqrt[-c + (d*e)/f]*f*h + (b^3*B*c*d^2*e*Sqrt[-
c + (d*e)/f]*g)/(c + d*x)^2 - (2*a*b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c
+ d*x)^2 - (a*b^2*B*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 + (2*a^2*b*C*d
^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (b^3*B*c^2*d*Sqrt[-c + (d*e)/f]*f
*g)/(c + d*x)^2 + (2*a*b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (
a*b^2*B*c*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 - (2*a^2*b*c*C*d^2*Sqrt[
-c + (d*e)/f]*f*g)/(c + d*x)^2 - (b^3*B*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c +
 d*x)^2 + (2*a*b^2*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*b^2*B*
c*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 - (2*a^2*b*c*C*d^2*e*Sqrt[-c + (
d*e)/f]*h)/(c + d*x)^2 + (b^3*B*c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 -
(2*a*b^2*c^3*C*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (a*b^2*B*c^2*d*Sqrt[-
c + (d*e)/f]*f*h)/(c + d*x)^2 + (2*a^2*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h)/(
c + d*x)^2 + (b^3*B*c*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (2*a*b^2*c*C*d
*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (a*b^2*B*d^2*Sqrt[-c + (d*e)/f]*f*g)/
(c + d*x) + (2*a^2*b*C*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (b^3*B*c*d*
e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (2*a*b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h)
/(c + d*x) - (a*b^2*B*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) + (2*a^2*b*...
```

3.20.3 Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2004, 2102, 2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2102}$$

$$\frac{\int \frac{2Cdfha^3-2b(Bdfh+C(dfg+deh+cfh))a^2+2b^2B(dfg+deh+cfh)a+2b(bB-2aC)dfhxa+b^2(bB-2aC)dfhx^2-b^3(Bdeg-c(2Ceg-Bfg-Beh))}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2(bc-ad)(be-af)(bg-ah)}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 2110$$

$$\frac{\int \frac{-2Cdfha^2+bBdfha+(b^2Bdfh-2abCdfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+\right.}{2(bc-ad)(be-af)(bg-ah)}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 176$$

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ce\right.}{2(bc-ad)(}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 124$$

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ce\right.}{2(bc}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 123$$

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ce\right.}{}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 131$$

3.20. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ce\cdots}{\cdots}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 131

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ce\cdots}{\cdots}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 130

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ce\cdots}{\cdots}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 187

$$\frac{-2\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2\cdots}{\cdots}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 413

$$\frac{-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ceg))\right)\int\frac{\cdots}{(bc-ad-b(c+dx))\sqrt{\frac{f(\cdots}{d\cdots}}}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}\cdots}{\cdots}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 413

3.20. $\displaystyle\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ceg)))\int \frac{}{(bc-ad}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow 412$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ceg)))}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[
e + f*x]*Sqrt[g + h*x]),x]
```

output
```
-((b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*
d)*(b*e - a*f)*(b*g - a*h)*(a + b*x))) + ((2*b*(b*B - 2*a*C)*Sqrt[f]*Sqrt[
-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcS
in[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
c*h))])/(Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*(b*B - 2*a*C)
*Sqrt[f]*Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(Sqrt[e + f*x]*Sqrt[g
+ h*x]) - (2*Sqrt[-(d*e) + c*f]*(4*a^3*C*d*f*h + 2*a*b^2*B*(d*f*g + d*e*h
+ c*f*h) - b^3*(B*d*e*g - c*(2*C*e*g - B*f*g - B*e*h)) - a^2*b*(3*B*d*f*h
+ 2*C*(d*f*g + d*e*h + c*f*h)))*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1
 + (h*(c + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)
), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*
(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sq
rt[g - (c*h)/d + (h*(c + d*x))/d]))/(2*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)
)
```

3.20. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.20.3.1 Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

3.20. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

3.20.4 Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 1211, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1211
default	Expression too large to display	13369

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2)/
(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(b^2/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*
g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*
x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)-a*d*f*h*(B*b-2*C*a)/(a^3*
d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*
e*g-b^3*c*e*g)*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1
/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*
e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-
g/h+e/f)/(-g/h+c/d))^(1/2))-d*f*h*b*(B*b-2*C*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2
*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(g/h-e
/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/
f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x
+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/
(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g
/h+c/d))^(1/2)))+(3*B*a^2*b*d*f*h-2*B*a*b^2*c*f*h-2*B*a*b^2*d*e*h-2*B*a*b^
2*d*f*g+B*b^3*c*e*h+B*b^3*c*f*g+B*b^3*d*e*g-4*C*a^3*d*f*h+2*C*a^2*b*c*f*h+
2*C*a^2*b*d*e*h+2*C*a^2*b*d*f*g-2*C*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*
b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(g/h-
e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e
/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e...
```

3.20.5 Fricas [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.20.6 Sympy [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**3/(d*x+c)**(1/2)/(f
*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Timed out
```

3.20.7 Maxima [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{(bx+a)^3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^3*sqrt(d*x + c)
*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.20.8 Giac [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{(bx+a)^3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^3*sqrt(d*x + c)
*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Hanged}$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^3*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.21 $\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.21.1 Optimal result

Integrand size = 62, antiderivative size = 980

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{b(4bBdfh+C(adfh-3b(dfg+deh+cfh)))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4df^2h^2\sqrt{c+dx}}$$
$$+ \frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{b\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh+C(adfh-3b(dfg+deh+cfh)))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\Big(\arcsin\Big(\frac{\sqrt{}}{\sqrt{}}}{4d^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+ \frac{(be-af)\sqrt{bg-ah}(aCdfh-b(4Bdfh-C(3dfg+3deh+cfh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\Big(\text{ar}}{4df^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$- \frac{\sqrt{-dg+ch}((adfh+b(dfg+deh+cfh))(4bBdfh+C(adfh-3b(dfg+deh+cfh)))+4dfh(2a^2Cdf}{}$$

output
```
-1/4*((a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*(4*b*B*d*f*h+C*(a*d*f*h-3*b*(c*f*h+d
*e*h+d*f*g)))+4*d*f*h*(2*a^2*C*d*f*h+b^2*C*(c*e*h+c*f*g+d*e*g)-a*b*(4*B*d*
f*h-C*(c*f*h+d*e*h+d*f*g))))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(
1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*
(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+
c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)
/d^2/f^2/h^3/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/4*b*(4*b*B*d*f
*h+C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g
)^(1/2)/d/f^2/h^2/(d*x+c)^(1/2)+1/2*b^2*C*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x
+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+1/4*(-a*f+b*e)*(a*C*d*f*h-b*(4*B*d*f*h-C*(c*
f*h+3*d*e*h+3*d*f*g)))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)
^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))
*(-a*h+b*g)^(1/2)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1
/2)/d/f^2/h^2/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*
g)/(b*x+a))^(1/2)-1/4*b*(4*b*B*d*f*h+C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g)))*
EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((
-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+
f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d^
2/f^2/h^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

3.21.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21961 vs. $2(980) = 1960$.

Time = 36.91 (sec) , antiderivative size = 21961, normalized size of antiderivative = 22.41

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x]*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

3.21.3 Rubi [A] (warning: unable to verify)

Time = 3.13 (sec) , antiderivative size = 978, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2004, 2100, 2105, 25, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\left(a^2(-C)+abB+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{(a+bx)^{3/2}\left(\frac{abB-a^2C}{a}+bCx\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2100}$$

$$\int \frac{4(bB-aC)dfha^2+b^2(4bBdfh+aCdfh-3bC(dfg+deh+cfh))x^2-b^2C(bceg+a(deg+cfg+ceh))-2b(2Cdfha^2-b(4Bdfh-C(dfg+deh+cfh))a+b}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}$$

$$4dfh$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{2105}$$

$$\int -\frac{b(b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh)))+b((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}$$

$$2bdfh$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{b(b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh)))+b((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}$$

$$2bdfh$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)+b\left((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 194

$$-\frac{\int \frac{b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)+b\left((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 327

$$-\frac{\int \frac{b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)+b\left((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 2101

$$-\frac{\left(4dfh\left(2a^2Cdfh-ab(4Bdfh-C(cfh+deh+dfg))+b^2C(ceh+cfg+deg)\right)+(adfh+b(cfh+deh+dfg))(aCdfh+4bBdfh-3bC(cfh+deh+dfg))\right)\int}{2dfh}$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 183

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}+$$

$$-\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{b\sqrt{a+bx}\sqrt{}}{}$$

$\downarrow$ 188

3.21. $\int \frac{\sqrt{a+bx}\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}+$$

$$-\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{b\sqrt{a+bx}\sqrt{}}{}$$

$\downarrow$ 321

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}+$$

$$-\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{b\sqrt{a+bx}\sqrt{}}{}$$

$\downarrow$ 412

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}+$$

$$-\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{b\sqrt{a+bx}\sqrt{}}{}$$

input
```
Int[(Sqrt[a + b*x]*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

3.21. $\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(b^2*C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h)
+ ((b*(4*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b
*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (b*Sqrt[d*g - c*h]*
Sqrt[f*g - e*h]*(4*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h))*
Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*Ell
ipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*
x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(d*f*h*Sqrt[((
d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - ((2*d*(b*e
 - a*f)*Sqrt[b*g - a*h]*(4*b*B*d*f*h - a*C*d*f*h - b*C*(c*f*h + 3*d*(f*g +
 e*h)))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x
]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a
 + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[
f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a +
b*x)))]) + (2*Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(4
*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h)) + 4*d*f*h*(2*a^2*C
*d*f*h + b^2*C*(d*e*g + c*f*g + c*e*h) - a*b*(4*B*d*f*h - C*(d*f*g + d*e*h
 + c*f*h))))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)
)]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(
d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt
[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*...
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

3.21. $\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(897) = 1794$.

Time = 5.28 (sec) , antiderivative size = 1834, normalized size of antiderivative = 1.87

method	result	size
elliptic	Expression too large to display	1834
default	Expression too large to display	56432

input
```
int((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(1/2*C*b^2/d/f/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^
3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+
b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(
1/2)+2*(a^2*b*B-C*a^3-1/2*C*b^2/d/f/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g
+1/2*b*c*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+
c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(
-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x
+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/
2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(2*a*b^2*B-C*a^2*b
-1/2*C*b^2/d/f/h*(a*c*f*h+a*d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(g/h-a
/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x
+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1
/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*
(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(
g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x
+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+(B*b^3+C*b^2*a-1/2*C*b^2/d/f/h*(3/2*a*d*f*
h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*b*d*f*g))*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)
*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x...
```

3.21.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.21.6 Sympy [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\,(Bb-Ca+Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)
*sqrt(g + h*x)), x)
```

3.21.7 Maxima [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*sqrt(b*x + a)/(sqrt(d*x +
c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.21.8 Giac [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*sqrt(b*x + a)/(sqrt(d*x +
c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\int \frac{\sqrt{a+b\,x}\,\left(-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x\right)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/
2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(1/2)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/
2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.22 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.22.1 Optimal result

Integrand size = 62, antiderivative size = 734

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}} - \frac{bC\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{C(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} - \frac{\sqrt{-dg+ch}(aCdfh-b(2Bdfh-C(dfg+deh+cfh)))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticP}}{d\sqrt{bc-ad}fh^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-(a*C*d*f*h-b*(2*B*d*f*h-C*(c*f*h+d*e*h+d*f*g)))*(b*x+a)*EllipticPi((-a*d+
b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d
+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/
2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h
+f*g)/(b*x+a))^(1/2)/d/f/h^2/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+
b*C*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/h/(d*x+c)^(1/2)-C*(-a*f+b*
e)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2)
,(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-a*h+b*g)^(1/2)*((
-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/f/h/(-e*h+f*g)^(
1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-b*C*Elli
pticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d
+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)
^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/d/f/h/
((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

3.22.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8107 vs. 2(734) = 1468.

Time = 42.83 (sec) , antiderivative size = 8107, normalized size of antiderivative = 11.04

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

3.22.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2004, 2099, 183, 188, 194, 321, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2004}$$

$$\int \frac{\sqrt{a+bx}\left(\frac{abB-a^2C}{a}+bCx\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2099}$$

$$\frac{(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}+\frac{bC(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{C(be-af)(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2fh}+\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 183}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\cdots}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}+\frac{bC(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{C(be-af)(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2fh}+\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 188}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\cdots}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}+\frac{bC(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{C\sqrt{g+hx}(be-af)(bg-ah)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{fh\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 194}$$

3.22. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int\frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{C\sqrt{g+hx}(be-af)(bg-ah)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int\frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{fh\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{bC\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int\frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow 321$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int\frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{bC\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int\frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}-$$

$$\frac{C\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+$$

$$\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow 327$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int\frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{C\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{bC\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow 412$$

3.22. $\int\frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\,\text{EllipticPi}\left(-\frac{b(dg-c}{(bc-ad}\right.}{dfh^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

$$\frac{C\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{bC\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

input

```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]),x]
```

output

```
(b*C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (b*C
*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*
x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f
*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a
*f)*(d*g - c*h))])/(d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d
*x))]*Sqrt[g + h*x]) - (C*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
 - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f
*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(f*h*Sqrt[f*g - e*h]*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[-(d*g)
+ c*h]*(2*b*B*d*f*h - a*C*d*f*h - b*C*(d*f*g + d*e*h + c*f*h))*(a + b*x)*S
qrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e
+ f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)
*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a +
b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(d*Sqrt[b*c
- a*d]*f*h^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.22. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.22.3.1 Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.22. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2099
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[B*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*Sqrt[c + d*x])), x] + (-Sim
p[B*(b*e - a*f)*((b*g - a*h)/(2*b*f*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x
]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Simp[B*(d*e - c*f)*((d*g - c*h)/(2
*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Simp[(2*A*b*d*f*h + B*(a*d*f*h - b*(d*f*g + d*e*h + c*f*h)))/(2*b
*d*f*h)   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && NeQ[2*A*d*f - B*(d*e +
 c*f), 0]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. 2(669) = 1338.

Time = 4.90 (sec) , antiderivative size = 1552, normalized size of antiderivative = 2.11

method	result	size
elliptic	Expression too large to display	1552
default	Expression too large to display	20101

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*(B*a*b-C*a^2)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h
+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/
2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*
d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)
/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/
2))+2*B*b^2*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d
)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/
h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/
f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^
(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*Ellipti
cPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((
e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+C*b^2*((x+a/b)*(x+e/f)*(
x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*(
(-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b
)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*
EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/
b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/
(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2
))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*Elli...
```

3.22.5 Fricas [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.22.6 Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}(Bb - Ca + Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(1/2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sq
rt(g + h*x)), x)
```

3.22.7 Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(b*x + a)*sqrt(d*x +
c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.22.8 Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(b*x + a)*sqrt(d*x +
c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.23 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.23.1 Optimal result

Integrand size = 62, antiderivative size = 436

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(bB-2aC)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}}{\sqrt{fg-e}}\right.\right.}{\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g}{(fg-eh)(}}}$$
$$+\frac{2C\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-af)(}{(bc-ad)(f}\right.}{\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2*C*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x
+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e
*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1
/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/h/(-a*d+b*c)^(1/2)/(d*x+
c)^(1/2)/(f*x+e)^(1/2)+2*(B*b-2*C*a)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1
/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*
h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)
/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*
h+f*g)/(b*x+a))^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 25.18 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.34

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(a+bx)^{3/2}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-\frac{bB\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\text{arcs}\right.}{(bg-ah)(a+bx)}\right.}{}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(3/2)*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(
-((b*B*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*Ell
ipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], (
(-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*(a
 + b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])) - (2*a*
C*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*Elliptic
F[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*
c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((-(b*g) + a*h)*(a +
 b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]) + (C*(-(f*
g) + e*h)*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h
)^2*(a + b*x)^2))]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a*f)*h), ArcSin[S
qrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*
(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*e - a*f)*h)))/(Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

3.23.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2004, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C) + abB + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \; 2004$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2101}$$

$$(bB-2aC)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+C\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{183}$$

$$(bB-2aC)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{188}$$

$$\frac{2\sqrt{g+hx}(bB-2aC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{g+hx}(bB-2aC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2\sqrt{g+hx}(bB-2aC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2C(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

3.23. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*(b*B - 2*a*C)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqr
t[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*
h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))
])/(Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g +
 h*x))/((f*g - e*h)*(a + b*x)))]) + (2*C*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt
[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f
*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)
), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x
])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(Sqrt[b*c - a*d
]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.23.3.1 Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.23. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(398) = 796$.

Time = 6.31 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.96

method	result
elliptic	$\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\left(\frac{2(Bb-Ca)\left(\frac{g}{h}-\frac{a}{b}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}F\left(\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)(}{\left(-\frac{g}{h}+\frac{a}{b}\right)(}}\right.}{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}}\right.$
default	Expression too large to display

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*(B*b-C*a)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b
)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*(
(-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*
h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g
/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+
2*C*b*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((
-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)
/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+
g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),
((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((
-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/
d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))
```

3.23.5 Fricas [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.23.6 Sympy [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bb-Ca+Cbx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(3/2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((B*b - C*a + C*b*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sq
rt(g + h*x)), x)
```

3.23.7 Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(3/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.23.8 Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(3/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

3.24 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.24.1 Optimal result

Integrand size = 62, antiderivative size = 616

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b(bB-2aC)d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}$$
$$-\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$
$$-\frac{2b(bB-2aC)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+\frac{2(bcC-bBd+aCd)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

output
```
2*b*(B*b-2*C*a)*d*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a
*f+b*e)/(-a*h+b*g)/(d*x+c)^(1/2)-2*b^2*(B*b-2*C*a)*(d*x+c)^(1/2)*(f*x+e)^(
1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*(-B*b*
d+C*a*d+C*b*c)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(
b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+
b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*h+b*g)
^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x
+a))^(1/2)-2*b*(B*b-2*C*a)*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+
f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/
2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(
-e*h+f*g)/(d*x+c))^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/((-c*f+d*e)*(b*x
+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 26.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.55

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left(b(bB-2aC\right.}{}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(5/2)*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
f*x)^(3/2)*(g + h*x)^(3/2)*(b*(b*B - 2*a*C)*(d*g - c*h)*EllipticE[ArcSin[S
qrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*
g - e*h))/((b*e - a*f)*(d*g - c*h))] + (b*c*C - b*B*d + a*C*d)*(b*g - a*h)
*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]
], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*
g - e*h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e +
f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

3.24.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 586, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2004, 2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2102}$$

$$\frac{\int \frac{2(bB-2aC)dfhx^2b^2+C(bceg-a(deg+cfg+ceh))b^2+(bB-2aC)(adfh+b(dfg+deh+cfh))xb-a(bB-aC)(adfh-b(dfg+deh+cfh))}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{(bc-ad)(be-af)(bg-ah)}-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{2105}$$

$$\frac{b(bB-2aC)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{\int \frac{2bd(bcC+adC-bBd)f(be-af)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{2bd\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+\cdots}}{\sqrt{c+dx}}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b(bB-2aC)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+(be-af)(bg-ah)(aCd-bBd+bcC)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\cdots}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{188}$$

$$\frac{b(bB-2aC)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2\sqrt{g+hx}(be-af)(bg-ah)(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-\cdots}{(de-\cdots}}}\cdots}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-\cdots}{(a+bx)(fg-\cdots}}}}{(bc-ad)(be-af)(bg-ah)}\cdots-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 194

$$\frac{\frac{2\sqrt{g+hx}(be-af)(bg-ah)(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{a+bx}(bB-2aC)(dg-ch)\sqrt{-\cdots}}{\sqrt{g+\cdots}}}{(bc-ad)(be-af)(bg-ah)}\cdots-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 321

$$\frac{-\frac{2b\sqrt{a+bx}(bB-2aC)(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{2\sqrt{g+hx}(be-af)\sqrt{bg-ah}(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\cdots}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\cdots}}}{(bc-ad)(be-af)(bg-ah)}\cdots-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 327

$$\frac{\frac{2\sqrt{g+hx}(be-af)\sqrt{bg-ah}(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{a+bx}(bB-2aC)\sqrt{dg-\cdots}}{\cdots}}{(bc-ad)(be-af)(bg-a\cdots}\cdots-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

3.24. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(-2*b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a
*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*b*(b*B - 2*a*C)*d*Sqrt[a
+ b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*b*(b*B - 2*a*C)*Sqr
t[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/
((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])
/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*
(d*g - c*h))])/(Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt
[g + h*x]) + (2*(b*c*C - b*B*d + a*C*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[Ar
cSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -((
(b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[f*g - e*h]*Sqr
t[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]))/((b*
c - a*d)*(b*e - a*f)*(b*g - a*h))
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.24. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2248 vs. $2(562) = 1124$.

Time = 7.51 (sec) , antiderivative size = 2249, normalized size of antiderivative = 3.65

method	result	size
elliptic	Expression too large to display	2249
default	Expression too large to display	18867

input

```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+
b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*
h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a)/(
(x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g
*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(C+(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g
+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(B*b-2*C*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d
*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b
*c*f*g+b*d*e*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e
*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a))*(g/h-a/b)*((-g/h+c/d)*(
x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/
(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-
c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+
c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c
/d+g/h))^(1/2))+2*(-b*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(B*b-2*C*a)/(a^3*d
*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e
*g-b^3*c*e*g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2
*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2
*C*a))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*(
(-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+...
```

3.24.5 Fricas [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
integral((C*b*x - C*a + B*b)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqr
t(h*x + g)/(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2
*a*b*d)*f)*h)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d
)*e + (2*a*b*c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2
*d)*f)*g + (a^2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f
+ (2*a*b*c + a^2*d)*e)*g)*x), x)
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(5/2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Timed out
```

3.24.7 Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(5/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.24.8 Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(5/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{(a+b\,x)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(5/2)*(c + d*x)^(1/2)), x)
```

3.25 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.25.1 Optimal result

Integrand size = 62, antiderivative size = 1128

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2bd(9a^3Cdfh-b^3(2Bdeg-c(3Ceg-2Bfg-2Beh))+ab^2 \ldots}{\ldots}$$

$$-\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2b^2(9a^3Cdfh-b^3(2Bdeg-c(3Ceg-2Bfg-2Beh))+ab^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh \ldots}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+ \ldots}}$$

$$-\frac{2b\sqrt{dg-ch}\sqrt{fg-eh}(9a^3Cdfh-b^3(2Bdeg-c(3Ceg-2Bfg-2Beh))+ab^2(C(deg+cfg+ceh)+4 \ldots}{3(bc-ad)^2(be- \ldots}$$

$$-\frac{2(3a^3Cd^2fh-b^3(2Bd^2eg-Bc^2fh-cd(3Ceg-Bfg-Beh))-3a^2bd(Bdfh+C(dfg+deh-cfh))+ \ldots}{3(bc-ad)^2(be-af)(\ldots}$$

output
```
2/3*b*d*(9*a^3*C*d*f*h-b^3*(2*B*d*e*g-c*(-2*B*e*h-2*B*f*g+3*C*e*g))+a*b^2*
(C*(c*e*h+c*f*g+d*e*g)+4*B*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*B*d*f*h+5*C*(c*f*
h+d*e*h+d*f*g)))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-
a*f+b*e)^2/(-a*h+b*g)^2/(d*x+c)^(1/2)-2/3*b^2*(B*b-2*C*a)*(d*x+c)^(1/2)*(f
*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)-2
/3*b^2*(9*a^3*C*d*f*h-b^3*(2*B*d*e*g-c*(-2*B*e*h-2*B*f*g+3*C*e*g))+a*b^2*(
C*(c*e*h+c*f*g+d*e*g)+4*B*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*B*d*f*h+5*C*(c*f*h
+d*e*h+d*f*g)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a
*f+b*e)^2/(-a*h+b*g)^2/(b*x+a)^(1/2)-2/3*(3*a^3*C*d^2*f*h-b^3*(2*B*d^2*e*g
-B*c^2*f*h-c*d*(-B*e*h-B*f*g+3*C*e*g))-3*a^2*b*d*(B*d*f*h+C*(-c*f*h+d*e*h+
d*f*g))+a*b^2*(3*B*d^2*(e*h+f*g)+C*(-2*c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)))*
EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-
(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-
c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)^(
3/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a
))^(1/2)-2/3*b*(9*a^3*C*d*f*h-b^3*(2*B*d*e*g-c*(-2*B*e*h-2*B*f*g+3*C*e*g))
+a*b^2*(C*(c*e*h+c*f*g+d*e*g)+4*B*(c*f*h+d*e*h+d*f*g))-a^2*b*(6*B*d*f*h+5*
C*(c*f*h+d*e*h+d*f*g)))*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g
)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))
*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(...
```

3.25.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10836 vs. $2(1128) = 2256$.

Time = 40.01 (sec) , antiderivative size = 10836, normalized size of antiderivative = 9.61

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(7/2)*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

3.25.3 Rubi [A] (warning: unable to verify)

Time = 4.57 (sec) , antiderivative size = 1105, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.145$, Rules used = {2004, 2102, 2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int \frac{C(3bceg-a(deg+cfg+ceh))b^2+(bB-2aC)(3adfh-b(dfg+deh+cfh))xb-(bB-aC)\left(3dfha^2-3b(dfg+deh+cfh)a+2b^2(deg+cfg+ceh)\right)}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3(bc-ad)(be-af)(bg-ah)}-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2102}$$

$$\int \frac{2dfh\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a+b^3(3cCeg-2Bdeg-2Bc(fg+eh))\right)x^2b^2+(bB-2aC)(bceg-a(deg\ldots}{\ldots}$$

$$\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2105}$$

$$\frac{b(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a+b^3(3cC\ldots\right.}{\ldots}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a+b^3(3cC\right.}{}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 188

$$\frac{b(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a+b^3(3cC\right.}{}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 194

$$-\frac{2b(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 321

$$-\frac{2b(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 327

$$-\frac{2b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

3.25. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c -
 a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) + ((-2*b^2*(9*a^3*C*d*f*h +
 b^3*(3*c*C*e*g - 2*B*d*e*g - 2*B*c*(f*g + e*h)) + a*b^2*(C*(d*e*g + c*f*g
 + c*e*h) + 4*B*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*B*d*f*h + 5*C*(d*f*g +
 d*e*h + c*f*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*
(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*b*d*(9*a^3*C*d*f*h + b^3*(3*c
*C*e*g - 2*B*d*e*g - 2*B*c*(f*g + e*h)) + a*b^2*(C*(d*e*g + c*f*g + c*e*h)
 + 4*B*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*B*d*f*h + 5*C*(d*f*g + d*e*h +
c*f*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*b*S
qrt[d*g - c*h]*Sqrt[f*g - e*h]*(9*a^3*C*d*f*h + b^3*(3*c*C*e*g - 2*B*d*e*g
 - 2*B*c*(f*g + e*h)) + a*b^2*(C*(d*e*g + c*f*g + c*e*h) + 4*B*(d*f*g + d*
e*h + c*f*h)) - a^2*b*(6*B*d*f*h + 5*C*(d*f*g + d*e*h + c*f*h)))*Sqrt[a +
b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[Ar
cSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b
*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[((d*e - c*f)*(a +
 b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[b*g -
 a*h]*(3*a^3*C*d^2*f*h - 3*a^2*b*d*(B*d*f*h + C*(d*f*g + d*e*h - c*f*h)) -
 b^3*(2*B*d^2*e*g - B*c^2*f*h - c*d*(3*C*e*g - B*(f*g + e*h))) + a*b^2*(3*
B*d^2*(f*g + e*h) + C*(d^2*e*g - 2*c^2*f*h - c*d*(f*g + e*h))))*Sqrt[((b*e
 - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[Arc...
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

3.25. $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3424 vs. $2(1056) = 2112$.

Time = 10.27 (sec) , antiderivative size = 3425, normalized size of antiderivative = 3.04

method	result	size
elliptic	Expression too large to display	3425
default	Expression too large to display	110289

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2/3/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+
a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a)*(b*d*f*h*x^4+a*
d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*
f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+
b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+2/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^
2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)*b/(a^3*d*f*h-a^2*b*c*
f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)
^2*(6*B*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*h-4*B*a*b^2*d*f*g+2*B*b^
3*c*e*h+2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+5*C*a^2*b*c*f*h+5*C*a^2*
b*d*e*h+5*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*g-C*a*b^2*d*e*g-3*C*b^3*
c*e*g)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x
+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(-1/3*(3*B*a*b*d*f*h-B*b^2*c*f*h-B*
b^2*d*e*h-B*b^2*d*f*g-6*C*a^2*d*f*h+2*C*a*b*c*f*h+2*C*a*b*d*e*h+2*C*a*b*d*
f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*
g+a*b^2*d*e*g-b^3*c*e*g)+1/3*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*
c*e*h+b^2*c*f*g+b^2*d*e*g)*(6*B*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*
h-4*B*a*b^2*d*f*g+2*B*b^3*c*e*h+2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+
5*C*a^2*b*c*f*h+5*C*a^2*b*d*e*h+5*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*
g-C*a*b^2*d*e*g-3*C*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b...
```

3.25.5 Fricas [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
integral((C*b*x - C*a + B*b)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqr
t(h*x + g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3
*a*b^2*d)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a
*b^2*d)*e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*
(a*b^2*c + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*
f)*h)*x^3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*
f + (3*a^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a
^3*d)*e)*g)*x), x)
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(7/2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Timed out
```

3.25.7 Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(7/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.25.8 Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(7/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{(a+b\,x)}^{7/2}\,\sqrt{c+d\,x}}\,dx$$

input

```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

output

```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(7/2)*(c + d*x)^(1/2)), x)
```

3.26 $\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.26.1 Optimal result

Integrand size = 42, antiderivative size = 1097

$$\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2(4C(2adfh-3b(dfg+deh+cfh))(adfh-2b(dfg+deh+cfh))+5bdfh(7Abdfh-C(5b(deg+cfg+}{105d^3f^3h^3}$$
$$+\frac{4C(2adfh-3b(dfg+deh+cfh))(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{35d^2f^2h^2}$$
$$+\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}$$
$$-\frac{4\sqrt{-de+cf}(35a^2Cd^2f^2h^2(dfg+deh+cfh)-7abdfh(15Ad^2f^2h^2+C(8c^2f^2h^2+7cdfh(fg+eh)+}{}$$
$$+\frac{2\sqrt{-de+cf}(35a^2d^2f^2h^2(3Adfh^2+C(ch(fg-eh)+dg(2fg+eh)))-14abdfh(15Ad^2f^2gh^2+C(4c^2}{}$$

output
```
2/105*(4*C*(2*a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(a*d*f*h-2*b*(c*f*h+d*e*h+d
*f*g))+5*b*d*f*h*(7*A*b*d*f*h-C*(5*b*(c*e*h+c*f*g+d*e*g)+2*a*(c*f*h+d*e*h+
d*f*g))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^3/f^3/h^3+4/35*C*(2*
a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+
g)^(1/2)/d^2/f^2/h^2+2/7*C*(b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(
1/2)/d/f/h-4/105*(35*a^2*C*d^2*f^2*h^2*(c*f*h+d*e*h+d*f*g)-7*a*b*d*f*h*(15
*A*d^2*f^2*h^2+C*(8*c^2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g
*h+8*f^2*g^2)))+b^2*(35*A*d^2*f^2*h^2*(c*f*h+d*e*h+d*f*g)+2*C*(12*c^3*f^3*
h^3+10*c^2*d*f^2*h^2*(e*h+f*g)+c*d^2*f*h*(10*e^2*h^2+9*e*f*g*h+10*f^2*g^2)
+2*d^3*(6*e^3*h^3+5*e^2*f*g*h^2+5*e*f^2*g^2*h+6*f^3*g^3))))*EllipticE(f^(1
/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-
d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^4/f^(7/2)/h^4/(f*x
+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/105*(35*a^2*d^2*f^2*h^2*(3*A*d*f*
h^2+C*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g)))-14*a*b*d*f*h*(15*A*d^2*f^2*g*h^2+C
*(4*c^2*f*h^2*(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2
*h^2+3*e*f*g*h+8*f^2*g^2)))+b^2*(35*A*d^2*f^2*h^2*(c*h*(-e*h+f*g)+d*g*(e*h
+2*f*g))+C*(24*c^3*f^2*h^3*(-e*h+f*g)+c^2*d*f*h^2*(-23*e^2*h^2+6*e*f*g*h+1
7*f^2*g^2)+2*c*d^2*h*(-12*e^3*h^3+3*e^2*f*g*h^2+e*f^2*g^2*h+8*f^3*g^3)+d^3
*g*(24*e^3*h^3+17*e^2*f*g*h^2+16*e*f^2*g^2*h+48*f^3*g^3))))*EllipticF(f^(1
/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c...
```

3.26.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.54 (sec) , antiderivative size = 1291, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2\left(-2d^2\sqrt{-c+\frac{de}{f}}(35a^2Cd^2f^2h^2(dfg+deh+cfh)-7abdfh(15Ad^2f^2h^2+C(8c^2f^2h^2+7cdfh(fg+eh)\right.}{}$$

input
```
Integrate[((a + b*x)^2*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
(2*(-2*d^2*Sqrt[-c + (d*e)/f]*(35*a^2*C*d^2*f^2*h^2*(d*f*g + d*e*h + c*f*h
) - 7*a*b*d*f*h*(15*A*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*
h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2*f^2*h^2*(d*
f*g + d*e*h + c*f*h) + 2*C*(12*c^3*f^3*h^3 + 10*c^2*d*f^2*h^2*(f*g + e*h)
+ c*d^2*f*h*(10*f^2*g^2 + 9*e*f*g*h + 10*e^2*h^2) + 2*d^3*(6*f^3*g^3 + 5*e
*f^2*g^2*h + 5*e^2*f*g*h^2 + 6*e^3*h^3))))*(e + f*x)*(g + h*x) + d^2*Sqrt[
-c + (d*e)/f]*f*h*(c + d*x)*(e + f*x)*(g + h*x)*(35*a^2*C*d^2*f^2*h^2 - 14
*a*b*C*d*f*h*(4*c*f*h + d*(4*f*g + 4*e*h - 3*f*h*x)) + b^2*(35*A*d^2*f^2*h
^2 + C*(24*c^2*f^2*h^2 + c*d*f*h*(23*f*g + 23*e*h - 18*f*h*x) + d^2*(24*e^
2*h^2 + e*f*h*(23*g - 18*h*x) + 3*f^2*(8*g^2 - 6*g*h*x + 5*h^2*x^2))))) -
(2*I)*(d*e - c*f)*h*(35*a^2*C*d^2*f^2*h^2*(d*f*g + d*e*h + c*f*h) - 7*a*b*
d*f*h*(15*A*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(
8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2*f^2*h^2*(d*f*g + d*e*
h + c*f*h) + 2*C*(12*c^3*f^3*h^3 + 10*c^2*d*f^2*h^2*(f*g + e*h) + c*d^2*f*
h*(10*f^2*g^2 + 9*e*f*g*h + 10*e^2*h^2) + 2*d^3*(6*f^3*g^3 + 5*e*f^2*g^2*h
 + 5*e^2*f*g*h^2 + 6*e^3*h^3))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c
+ d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (
d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*d*h*(35*a^2*d
^2*f^2*h^2*(3*A*d*f^2*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) - 14
*a*b*d*f*h*(15*A*d^2*e*f^2*h^2 + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*...
```

3.26.3 Rubi [A] (verified)

Time = 3.29 (sec) , antiderivative size = 1112, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.262$, Rules used = {2104, 25, 2103, 2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2104}$$

$$\frac{\int -\frac{(a+bx)\left(-2C(2adfh-3b(dfg+deh+cfh))x^2-(7Abdfh-5bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+4bcCeg-7aAdfh+aC(deg+cfg+ceh)\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{7dfh}$$

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$
$$\frac{\int \frac{(a+bx)\left(-2C(2adfh-3b(dfg+deh+cfh))x^2-(7Abdfh-5bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+4bcCeg-7aAdfh+aC(deg+cfg+ceh)\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}d}{7dfh}$$

$\downarrow$ 2103

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$
$$\int -\frac{\left((4C(2adfh-3b(dfg+deh+cfh))(adfh-2b(dfg+deh+cfh))+5bdfh(7Abdfh-5bC(deg+cfg+ceh)-2aC(dfg+deh+cfh)))x^2\right)+2\left(C(3b(deg+cfg+ceh)+2a(dfg+\right.}{}$$

$\downarrow$ 2118

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$
$$2\int -\frac{d\left(-\left(\left(35Ad^2f^2(deg+cfg+ceh)h^2+C\left(24f^2h^2(fg+eh)c^3+dfh\left(23f^2g^2+34efhg+23e^2h^2\right)c^2+2d^2\left(12f^3g^3+17ef^2hg^2+17e^2fh^2g+12e^3h^3\right)c+d^3eg\left(24f^2g^2\right.\right.\right.\right.\right.}{}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$
$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\int \frac{-\left(\left(35Ad^2f^2(deg+c\right.\right.}{}}{}$$

$\downarrow$ 176

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$
$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-eh)\right.\right.}{}$$

$\downarrow$ 124

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$
$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-eh)\right.\right.}{}$$

3.26. $\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$\downarrow$ 123

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-eh)\right.\right.}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-eh)\right.\right.}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-eh)\right.\right.}{}$$

$\downarrow$ 130

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{2\sqrt{cf-de}\left(\left(35Ad^2f^2(c\right.\right.}{}$$

input
```
Int[((a + b*x)^2*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

3.26. $\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(7*d*f*h) - ((
-4*C*(2*a*d*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*(a + b*x)*Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) + ((-2*(35*A*b^2*d*f*h + 8*a^2*C*d*f*h
 - 25*b^2*C*(d*e*g + c*f*g + c*e*h) - 38*a*b*C*(d*f*g + d*e*h + c*f*h) + (
24*b^2*C*(d*f*g + d*e*h + c*f*h)^2)/(d*f*h))*Sqrt[c + d*x]*Sqrt[e + f*x]*S
qrt[g + h*x])/3 - ((-4*Sqrt[-(d*e) + c*f]*(35*a^2*C*d^2*f^2*h^2*(d*f*g + d
*e*h + c*f*h) - 7*a*b*d*f*h*(15*A*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2 + 7*c*d*f
*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2
*f^2*h^2*(d*f*g + d*e*h + c*f*h) + 2*C*(12*c^3*f^3*h^3 + 10*c^2*d*f^2*h^2*
(f*g + e*h) + c*d^2*f*h*(10*f^2*g^2 + 9*e*f*g*h + 10*e^2*h^2) + 2*d^3*(6*f
^3*g^3 + 5*e*f^2*g^2*h + 5*e^2*f*g*h^2 + 6*e^3*h^3))))*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-
(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x
]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(35*a^2*d^2*f^2
*h^2*(3*A*d*f*h^2 + c*C*h*(f*g - e*h) + C*d*g*(2*f*g + e*h)) - 14*a*b*d*f*
h*(15*A*d^2*f^2*g*h^2 + C*(4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 + e*
f*g*h - 4*e^2*h^2) + d^2*g*(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))) + b^2*(35
*A*d^2*f^2*h^2*(c*h*(f*g - e*h) + d*g*(2*f*g + e*h)) + C*(24*c^3*f^2*h^3*(
f*g - e*h) + c^2*d*f*h^2*(17*f^2*g^2 + 6*e*f*g*h - 23*e^2*h^2) + 2*c*d^2*h
*(8*f^3*g^3 + e*f^2*g^2*h + 3*e^2*f*g*h^2 - 12*e^3*h^3) + d^3*g*(48*f^3...
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

3.26. $\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2104
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f
*g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
 c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
 1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.26.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 1238, normalized size of antiderivative = 1.13

method	result	size
elliptic	Expression too large to display	1238
default	Expression too large to display	12279

input
```
int((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2/7*C*b^2/d/f/h*x^2*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)+2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d
*f*g))/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*
e*g*x+c*e*g)^(1/2)+2/3*(b^2*A+C*a^2-2/7*C*b^2/d/f/h*(5/2*c*e*h+5/2*c*f*g+5
/2*d*e*g)-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*(2
*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*
e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)+2*(a^2*A-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(
3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*c*e*g-2/3*(b^2*A+C*a^2-2/7*C*b^2/d/f/h*(5/
2*c*e*h+5/2*c*f*g+5/2*d*e*g)-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h
+3*d*f*g))/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2
*d*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((
x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+
c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/
f)/(-g/h+c/d))^(1/2))+2*(2*a*b*A-4/7*C*b^2/d/f/h*c*e*g-2/5*(2*C*a*b-2/7*C*
b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/2*d*e*g)
-2/3*(b^2*A+C*a^2-2/7*C*b^2/d/f/h*(5/2*c*e*h+5/2*c*f*g+5/2*d*e*g)-2/5*(2*C
*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*(2*c*f*h+2*d*e*h+2*d
*f*g))/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+
c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+...
```

3.26.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 1665, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^2\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="fricas")
```

output
```
2/315*(3*(15*C*b^2*d^4*f^4*h^4*x^2 + 24*C*b^2*d^4*f^4*g^2*h^2 + (23*C*b^2*
d^4*e*f^3 + (23*C*b^2*c*d^3 - 56*C*a*b*d^4)*f^4)*g*h^3 + (24*C*b^2*d^4*e^2
*f^2 + (23*C*b^2*c*d^3 - 56*C*a*b*d^4)*e*f^3 + (24*C*b^2*c^2*d^2 - 56*C*a*
b*c*d^3 + 35*(C*a^2 + A*b^2)*d^4)*f^4)*h^4 - 6*(3*C*b^2*d^4*f^4*g*h^3 + (3
*C*b^2*d^4*e*f^3 + (3*C*b^2*c*d^3 - 7*C*a*b*d^4)*f^4)*h^4)*x)*sqrt(d*x + c
)*sqrt(f*x + e)*sqrt(h*x + g) + (48*C*b^2*d^4*f^4*g^4 + 16*(C*b^2*d^4*e*f^
3 + (C*b^2*c*d^3 - 7*C*a*b*d^4)*f^4)*g^3*h + (11*C*b^2*d^4*e^2*f^2 + 14*(C
*b^2*c*d^3 - 3*C*a*b*d^4)*e*f^3 + (11*C*b^2*c^2*d^2 - 42*C*a*b*c*d^3 + 70*
(C*a^2 + A*b^2)*d^4)*f^4)*g^2*h^2 + (16*C*b^2*d^4*e^3*f + 14*(C*b^2*c*d^3
- 3*C*a*b*d^4)*e^2*f^2 + 7*(2*C*b^2*c^2*d^2 - 6*C*a*b*c*d^3 + 5*(C*a^2 + A
*b^2)*d^4)*e*f^3 + (16*C*b^2*c^3*d - 42*C*a*b*c^2*d^2 - 210*A*a*b*d^4 + 35
*(C*a^2 + A*b^2)*c*d^3)*f^4)*g*h^3 + (48*C*b^2*d^4*e^4 + 16*(C*b^2*c*d^3 -
 7*C*a*b*d^4)*e^3*f + (11*C*b^2*c^2*d^2 - 42*C*a*b*c*d^3 + 70*(C*a^2 + A*b
^2)*d^4)*e^2*f^2 + (16*C*b^2*c^3*d - 42*C*a*b*c^2*d^2 - 210*A*a*b*d^4 + 35
*(C*a^2 + A*b^2)*c*d^3)*e*f^3 + (48*C*b^2*c^4 - 112*C*a*b*c^3*d - 210*A*a*
b*c*d^3 + 315*A*a^2*d^4 + 70*(C*a^2 + A*b^2)*c^2*d^2)*f^4)*h^4)*sqrt(d*f*h
)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^
2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e
*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2
+ (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3...
```

3.26.6 Sympy [F]

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)(a+bx)^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**2*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

output
```
Integral((A + C*x**2)*(a + b*x)**2/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h
*x)), x)
```

3.26.7 Maxima [F]

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

3.26.8 Giac [F]

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(C\,x^2+A)\,(a+b\,x)^2}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + C*x^2)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(
1/2)),x)
```

output
```
int(((A + C*x^2)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(
1/2)), x)
```

3.27 $\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.27.1 Optimal result

Integrand size = 40, antiderivative size = 611

$$\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{4C(adfh-2b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$-\frac{2\sqrt{-de+cf}(10aCdfh(dfg+deh+cfh)-b(15Ad^2f^2h^2+C(8c^2f^2h^2+7cdfh(fg+eh)+d^2(8f^2g^2+}{15d^3f^{5/2}h^3\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{2\sqrt{-de+cf}(5adfh(3Adfh^2+C(ch(fg-eh)+dg(2fg+eh)))-b(15Ad^2f^2gh^2+C(4c^2fh^2(fg-eh}{15d}$$

output
```
4/15*C*(a*d*f*h-2*b*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+
g)^(1/2)/d^2/f^2/h^2+2/5*C*(b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/
2)/d/f/h-2/15*(10*a*C*d*f*h*(c*f*h+d*e*h+d*f*g)-b*(15*A*d^2*f^2*h^2+C*(8*c
^2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f^2*g^2))))*Elli
pticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1
/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d^3/f^(5/2
)/h^3/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/15*(5*a*d*f*h*(3*A*d*f*
h^2+C*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g)))-b*(15*A*d^2*f^2*g*h^2+C*(4*c^2*f*h
^2*(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2*h^2+3*e*f*
g*h+8*f^2*g^2))))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d
*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d
*(h*x+g)/(-c*h+d*g))^(1/2)/d^3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.27.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.26 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(-d^2\sqrt{-c+\frac{de}{f}}(15Abd^2f^2h^2-10aCdfh(dfg+deh+cfh)+bC(8c^2f^2h^2+7cdfh(fg+eh)+d^2(8f\right.}{}$$

input
```
Integrate[((a + b*x)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
(-2*(-(d^2*Sqrt[-c + (d*e)/f]*(15*A*b*d^2*f^2*h^2 - 10*a*C*d*f*h*(d*f*g +
d*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g
^2 + 7*e*f*g*h + 8*e^2*h^2)))*(e + f*x)*(g + h*x)) + C*d^2*Sqrt[-c + (d*e)
/f]*f*h*(c + d*x)*(e + f*x)*(g + h*x)*(4*b*c*f*h - 5*a*d*f*h + b*d*(4*f*g
+ 4*e*h - 3*f*h*x)) - I*(d*e - c*f)*h*(15*A*b*d^2*f^2*h^2 - 10*a*C*d*f*h*(
d*f*g + d*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*
(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2)))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(
f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-
c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*d*h*(5*a
*d*f*h*(3*A*d*f^2*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) - b*(15*
A*d^2*e*f^2*h^2 + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*(-4*f^2*g^2 + e*f*
g*h + 3*e^2*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^
(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*
EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*
e*h - c*f*h)]))/(15*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqrt[c + d*x]*Sqrt[e +
f*x]*Sqrt[g + h*x])
```

3.27.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2104, 25, 2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2104}$$

$$\frac{\int -\frac{-2C(adfh-2b(dfg+deh+cfh))x^2-(5Abdfh-3bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+2bcCeg-5aAdfh+aC(deg+cfg+ceh)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{5dfh}+\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \text{25}$$

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-\frac{\int \frac{-2C(adfh-2b(dfg+deh+cfh))x^2-(5Abdfh-3bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+2bcCeg-5aAdfh+aC(deg+cfg+ceh)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{5dfh}$$

$\Big\downarrow$ 2118

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-$$

$$\frac{2\int -\frac{d\left(5adfh(3Adfh-C(deg+cfg+ceh))+2bC\left(2fh(fg+eh)c^2+d\left(2f^2g^2+3efhg+2e^2h^2\right)c+2d^2eg(fg+eh)\right)+\left(15Abd^2f^2h^2-10aCdf(dfg+deh+cfh)h+bC\left(\left(8f^2\right.\right.\right.}{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3d^2fh}$$

$$5dfh$$

$\Big\downarrow$ 27

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-$$

$$-\frac{\int \frac{5adfh(3Adfh-C(deg+cfg+ceh))+2bC\left(2fh(fg+eh)c^2+d\left(2f^2g^2+3efhg+2e^2h^2\right)c+2d^2eg(fg+eh)\right)+\left(15Abd^2f^2h^2-10aCdf(dfg+deh+cfh)h+bC\left(\left(8f^2g^2+\right.\right.\right.}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3dfh}$$

$$5dfh$$

$\Big\downarrow$ 176

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-$$

$$-\frac{\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-eh)+cdh\left(-4e^2h^2+efgh+3f^2g^2\right)+d^2g\left(4e^2h^2+3efgh+8f^2g^2\right)\right)\right)\right)\int \frac{}{\sqrt{c+dx}\ldots}}{h}$$

$$3dfh$$

$\Big\downarrow$ 124

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-$$

$$-\frac{\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-eh)+cdh\left(-4e^2h^2+efgh+3f^2g^2\right)+d^2g\left(4e^2h^2+3efgh+8f^2g^2\right)\right)\right)\right)\int \frac{}{\sqrt{c+dx}\ldots}}{h}$$

$\Big\downarrow$ 123

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-$$

$$-\frac{\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-eh)+cdh\left(-4e^2h^2+efgh+3f^2g^2\right)+d^2g\left(4e^2h^2+3efgh+8f^2g^2\right)\right)\right)\right)\int \frac{}{\sqrt{c+dx}\ldots}}{h}$$

$\Big\downarrow$ 131

3.27. $\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} -$$

$$-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\Big(5adfh\big(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\big)-b\Big(15Ad^2f^2gh^2+C\big(4c^2fh^2(fg-eh)+cdh(-4e^2h^2+efgh+3f^2g^2)+d^2g(4e^2h^2+3efgh+8f^2g^2)\big)\Big)}{h\sqrt{e+fx}}$$

$\downarrow$ 131

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} -$$

$$-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Big(5adfh\big(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\big)-b\Big(15Ad^2f^2gh^2+C\big(4c^2fh^2(fg-eh)+cdh(-4e^2h^2+efgh+3f^2g^2)+d^2g(4e^2h^2+3efgh+}{h\sqrt{e+fx}\sqrt{g+hx}}$$

$\downarrow$ 130

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh} -$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)\Big(5adfh\big(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\big)-b\Big(15Ad^2f^2gh^2+C\big(4c^2fh^2(fg-eh}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

input
```
Int[((a + b*x)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*C*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) - ((-4
*C*(a*d*f*h - 2*b*(d*f*g + d*e*h + c*f*h))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x])/(3*d*f*h) - ((2*Sqrt[-(d*e) + c*f]*(15*A*b*d^2*f^2*h^2 - 10*a*
C*d*f*h*(d*f*g + d*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*
h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2)))*Sqrt[(d*(e + f*x))/(d*e - c
*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) +
c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(5*a*d*f*h*(3*A*d*f*h^2
 + c*C*h*(f*g - e*h) + C*d*g*(2*f*g + e*h)) - b*(15*A*d^2*f^2*g*h^2 + C*(4
*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 + e*f*g*h - 4*e^2*h^2) + d^2*g*(
8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[
-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*
x]*Sqrt[g + h*x]))/(3*d*f*h))/(5*d*f*h)
```

3.27. $\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2104
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f
*g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
 c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
 1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.27.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2Cbx\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(Ca-\frac{2Cb(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{dfh\,x^3+cfh\,x^2}}{3dfh}\right.}{}$
default	Expression too large to display

input

```
int((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_
RETURNVERBOSE)
```

output

```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2/5*C*b/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f
/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
^(1/2)+2*(A*a-2/5*C*b/d/f/h*c*e*g-2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2*d*e*h+
2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e
/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^
3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*Ellip
ticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))+2*(A*b-2/5*C
*b/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/2*d*e*g)-2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2
*d*e*h+2*d*f*g))/d/f/h*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^
(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f
*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d
)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*E
llipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

3.27.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1068, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\left(3\left(3\,Cbd^3f^3h^3x-4\,Cbd^3f^3gh^2-(4\,Cbd^3ef^2+(4\,Cbcd^2-5\,Cad^3)f^3)h^3\right)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}-\right.}{}$$

input
```
integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="fricas")
```

output
```
2/45*(3*(3*C*b*d^3*f^3*h^3*x - 4*C*b*d^3*f^3*g*h^2 - (4*C*b*d^3*e*f^2 + (4
*C*b*c*d^2 - 5*C*a*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
) - (8*C*b*d^3*f^3*g^3 + (3*C*b*d^3*e*f^2 + (3*C*b*c*d^2 - 10*C*a*d^3)*f^3
)*g^2*h + (3*C*b*d^3*e^2*f + (3*C*b*c*d^2 - 5*C*a*d^3)*e*f^2 + (3*C*b*c^2*
d - 5*C*a*c*d^2 + 15*A*b*d^3)*f^3)*g*h^2 + (8*C*b*d^3*e^3 + (3*C*b*c*d^2 -
 10*C*a*d^3)*e^2*f + (3*C*b*c^2*d - 5*C*a*c*d^2 + 15*A*b*d^3)*e*f^2 + (8*C
*b*c^3 - 10*C*a*c^2*d + 15*A*b*c*d^2 - 45*A*a*d^3)*f^3)*h^3)*sqrt(d*f*h)*w
eierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 -
 c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^
2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (
2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3),
 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)) - 3*(8*C*b*d^3*f^3*g^2*h
 + (7*C*b*d^3*e*f^2 + (7*C*b*c*d^2 - 10*C*a*d^3)*f^3)*g*h^2 + (8*C*b*d^3*e
^2*f + (7*C*b*c*d^2 - 10*C*a*d^3)*e*f^2 + (8*C*b*c^2*d - 10*C*a*c*d^2 + 15
*A*b*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*
f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27
*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2
*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2
*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*
e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), ...
```

3.27.6 Sympy [F]

$$\int \frac{(a+bx)\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)\,(a+bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),
x)
```

output
```
Integral((A + C*x**2)*(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)
), x)
```

3.27.7 Maxima [F]

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cx^2+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((C*x^2 + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

3.27.8 Giac [F]

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cx^2+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((C*x^2 + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\left(C\,x^2+A\right)\,\left(a+b\,x\right)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + C*x^2)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int(((A + C*x^2)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/
2)), x)
```

3.28 $\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.28.1 Optimal result

Integrand size = 35, antiderivative size = 368

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \frac{4C\sqrt{-de+cf}(dfg+deh+cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{2\sqrt{-de+cf}(3Adfh^2+C(ch(fg-eh)+dg(2fg+eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2/3*C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-4/3*C*(c*f*h+d*e*h+d
*f*g)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*
h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/
d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/3*(3*A*d*f*h^
2+C*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g)))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f
-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/
(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d^2/f^(3/2)/h^2/(f*x+e)^(1/
2)/(h*x+g)^(1/2)
```

3.28.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.77 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.06

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{\sqrt{c+dx}\left(2Cd^2fh(e+fx)(g+hx)-\frac{4Cd^2(dfg+deh+cfh)(e+fx)(g+hx)}{c+dx}-4iC\sqrt{-c+\frac{de}{f}}fh(dfg+deh+cfh)\right.}{}$$

input
```
Integrate[(A + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(Sqrt[c + d*x]*(2*C*d^2*f*h*(e + f*x)*(g + h*x) - (4*C*d^2*(d*f*g + d*e*h
+ c*f*h)*(e + f*x)*(g + h*x))/(c + d*x) - (4*I)*C*Sqrt[-c + (d*e)/f]*f*h*(
d*f*g + d*e*h + c*f*h)*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqr
t[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt
[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + ((2*I)*d*h*(3*A*d*f^2*h + c
*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x)
)/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqr
t[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[-c
+ (d*e)/f]))/(3*d^3*f^2*h^2*Sqrt[e + f*x]*Sqrt[g + h*x])
```

3.28.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$\Big\downarrow \; 2118$$
$$\frac{2\int \frac{d(3Adfh-C(deg+cfg+ceh)-2C(dfg+deh+cfh)x)}{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3d^2fh}+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3Adfh-C(deg+cfg+ceh)-2C(dfg+deh+cfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3dfh}+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{176}$$

$$\frac{\frac{(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{2C(cfh+deh+dfg)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}}{3dfh}+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{124}$$

$$\frac{\frac{(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{2C\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}(cfh+deh+dfg)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}}{3dfh}+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{123}$$

$$\frac{\frac{(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(cfh+deh+dfg)E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf}{f(dg-c}\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}}{3dfh}$$

$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{131}$$

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(cfh+deh+dfg)E\left(\arcsin\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}}{3dfh}$$

$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{131}$$

3.28. $\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(cfh-}{d\sqrt{f}h\sqrt{}}}{3dfh}$$

$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{130}$$

$$\frac{\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}}{d}}{3dfh}$$

$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

input
```
Int[(A + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*C*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((-4*C*Sqrt[-(
d*e) + c*f]*(d*f*g + d*e*h + c*f*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g
 + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*
e - c*f)*h)/(f*(d*g - c*h))]])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x)
)/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*A*d*f*h^2 + c*C*h*(f*g - e*h) +
 C*d*g*(2*f*g + e*h))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(
d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]],
((d*e - c*f)*h)/(f*(d*g - c*h))]])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]
))/(3*d*f*h)
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.28. $\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.28.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.66

method	result
elliptic	$\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2C\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(A-\frac{2C\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{}}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x}}\right.$
default	Expression too large to display

input
```
int((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2/3*C/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*
g*x+c*e*g)^(1/2)+2*(A-2/3*C/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(g/h-e/
f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f
))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+
c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(
1/2))-4/3*C/d/f/h*(c*f*h+d*e*h+d*f*g)*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*
((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2
+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*Elli
pticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*Ellipti
cF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

3.28.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 775, normalized size of antiderivative = 2.11

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\frac{2\left(3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}Cd^2f^2h^2+(2\,Cd^2f^2g^2+(Cd^2ef+Ccdf^2)gh+(2\,Cd^2e^2+Ccdef+(2\,Cc\right.}{}$$

input
```
integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="fricas")
```

output
```
2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*C*d^2*f^2*h^2 + (2*C*d^2*
f^2*g^2 + (C*d^2*e*f + C*c*d*f^2)*g*h + (2*C*d^2*e^2 + C*c*d*e*f + (2*C*c^
2 + 9*A*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 -
(d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2)
, -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f -
4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*
f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*
h)/(d*f*h)) + 6*(C*d^2*f^2*g*h + (C*d^2*e*f + C*c*d*f^2)*h^2)*sqrt(d*f*h)*
weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*
d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 +
 c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d
^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), we
ierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 -
c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2
 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2
*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3),
1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h))))/(d^3*f^3*h^3)
```

3.28.6 Sympy [F]

$$\int \frac{A + Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{A + Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx$$

input
```
integrate((C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((A + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.28.7 Maxima [F]

$$\int \frac{A + Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cx^2 + A}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((C*x^2 + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.28.8 Giac [F]

$$\int \frac{A + Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{Cx^2 + A}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="giac")
```

output
```
integrate((C*x^2 + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{C\,x^2 + A}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,\sqrt{c + d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.29 $\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.29.1 Optimal result

Integrand size = 42, antiderivative size = 465

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2C\sqrt{-de+cf}(bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\left(A+\frac{a^2C}{b^2}\right)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\mathrm{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*C*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/b/
d/h/f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*C*(a*h+b*g)*Ellip
ticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/
2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1
/2)/b^2/d/h/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(A+a^2*C/b^2)*EllipticPi
(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d
*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d
*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.29.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.55 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.23

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(b^2cCd^2e\sqrt{-c+\frac{de}{f}}g-abCd^3e\sqrt{-c+\frac{de}{f}}g-b^2c^2Cd\sqrt{-c+\frac{de}{f}}fg+abcCd^2\sqrt{-c+\frac{de}{f}}fg-b^2c^2Cde\sqrt{\ldots}\right.}{}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(-2*(b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g - a*b*C*d^3*e*Sqrt[-c + (d*e)/f]*g
 - b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g + a*b*c*C*d^2*Sqrt[-c + (d*e)/f]*f*g
 - b^2*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h + a*b*c*C*d^2*e*Sqrt[-c + (d*e)/f]*h
 + b^2*c^3*C*Sqrt[-c + (d*e)/f]*f*h - a*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h +
 b^2*c*C*d*Sqrt[-c + (d*e)/f]*f*g*(c + d*x) - a*b*C*d^2*Sqrt[-c + (d*e)/f]
*f*g*(c + d*x) + b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h*(c + d*x) - a*b*C*d^2*e*
Sqrt[-c + (d*e)/f]*h*(c + d*x) - 2*b^2*c^2*C*Sqrt[-c + (d*e)/f]*f*h*(c + d
*x) + 2*a*b*c*C*d*Sqrt[-c + (d*e)/f]*f*h*(c + d*x) + b^2*c*C*Sqrt[-c + (d*
e)/f]*f*h*(c + d*x)^2 - a*b*C*d*Sqrt[-c + (d*e)/f]*f*h*(c + d*x)^2 + I*b*C
*(b*c - a*d)*(d*e - c*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x)
)]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f
]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*b*d*(b*c*C*e - a*C*
d*e + a*c*C*f + A*b*d*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x)
)]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f
]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*A*b^2*d^2*f*h*(c +
d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*
x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d
*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*a^2*C*d^2*f*h*
(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt...
```

3.29.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2110}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\int \frac{\frac{Cx}{b}-\frac{aC}{b^2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{176}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{C\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{bh}$$

$$\Big\downarrow \text{124}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{C\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{bh\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{123}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{131}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b^2h\sqrt{e+fx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{131}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b^2h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{130}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{187}$$

$$-2\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{413}$$

3.29. $\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$

$$-\frac{2\left(\frac{a^2C}{b^2}+A\right)\sqrt{\frac{f(c+dx)}{de-cf}+1}\int\frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2\left(\frac{a^2C}{b^2}+A\right)\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int\frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2\left(\frac{a^2C}{b^2}+A\right)\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

input
```
Int[(A + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

3.29. $\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

output
```
(2*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Elli
pticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/
(f*(d*g - c*h))])/(b*d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c
*h)]) - (2*C*Sqrt[-(d*e) + c*f]*(b*g + a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)
]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])
/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*d*Sqrt[f]*h*S
qrt[e + f*x]*Sqrt[g + h*x]) - (2*(A + (a^2*C)/b^2)*Sqrt[-(d*e) + c*f]*Sqrt
[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1 + (h*(c + d*x))/(d*g - c*h)]*Ellipt
icPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*
Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sqrt[g - (c*h)/d + (h*(c + d*x))/d])
```

3.29.3.1 Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

3.29. $\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.29.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.61

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(-\frac{2Ca\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2C\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}}{b\sqrt{dfh\,x^3+cfh\,x}}\right.}{\sqrt{dx}}$
default	Expression too large to display

input
```
int((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(-2*C*a/b^2*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)
*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h
*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h
+e/f)/(-g/h+c/d))^(1/2))+2*C/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d
)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*
x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE((
(x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+
g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2)))+2*(A*b^2+C*a^2)/b^3*
(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-
g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d
*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*EllipticPi(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+
e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^(1/2)))
```

3.29.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="fricas")
```

output
```
Timed out
```

3.29.6 Sympy [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+Cx^2}{(a+bx)\,\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),
x)
```

output
```
Integral((A + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)
), x)
```

3.29.7 Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

3.29.8 Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((C*x^2 + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{C\,x^2 + A}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,(a + b\,x)\,\sqrt{c + d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)
),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)
), x)
```

3.30 $\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.30.1 Optimal result

Integrand size = 42, antiderivative size = 738

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{\left(Ab+\frac{a^2C}{b}\right)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{\sqrt{-de+cf}(a^2Cdf-2abC(de+cf)+b^2(2cCe-Adf))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right.\right.}{b^2d(bc-ad)\sqrt{f}(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{\sqrt{-de+cf}(a^4Cdfh-Ab^4(deg+cfg+ceh)-2a^3bC(dfg+deh+cfh)-2ab^3(2cCeg-Adfg-Ade}{b^2(bc-ad)^2\sqrt{f}(b}$$

output
```
-(A*b^2+C*a^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+
b*e)/(-a*h+b*g)/(b*x+a)+(A*b+a^2*C/b)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f
-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(c*f-d*e)^(1/2)*(d*
(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(
f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+(a^2*C*d*f-2*a*b*C*(c*f+d*e)+b^2
*(-A*d*f+2*C*c*e))*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+
d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(
d*(h*x+g)/(-c*h+d*g))^(1/2)/b^2/d/(-a*d+b*c)/(-a*f+b*e)/f^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2)-(a^4*C*d*f*h-A*b^4*(c*e*h+c*f*g+d*e*g)-2*a^3*b*C*(c*f*h+
d*e*h+d*f*g)-2*a*b^3*(-A*c*f*h-A*d*e*h-A*d*f*g+2*C*c*e*g)-3*a^2*b^2*(A*d*f
*h-C*(c*e*h+c*f*g+d*e*g)))*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2
),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^
(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b^2/(-a*d+
b*c)^2/(-a*f+b*e)/(-a*h+b*g)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

3.30.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.65 (sec) , antiderivative size = 3935, normalized size of antiderivative = 5.33

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
((-(A*b^2) - a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*(a + b*x)) - ((c + d*x)^(3/2)*(A*b^4*c*Sqrt[-c +
 (d*e)/f]*f*h + a^2*b^2*c*C*Sqrt[-c + (d*e)/f]*f*h - a*A*b^3*d*Sqrt[-c + (
d*e)/f]*f*h - a^3*b*C*d*Sqrt[-c + (d*e)/f]*f*h + (A*b^4*c*d^2*e*Sqrt[-c +
(d*e)/f]*g)/(c + d*x)^2 + (a^2*b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c + d*
x)^2 - (a*A*b^3*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (a^3*b*C*d^3*e*S
qrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (A*b^4*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c
 + d*x)^2 - (a^2*b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a*A*b^
3*c*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a^3*b*c*C*d^2*Sqrt[-c + (d*
e)/f]*f*g)/(c + d*x)^2 - (A*b^4*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2
- (a^2*b^2*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*A*b^3*c*d^2*e*
Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a^3*b*c*C*d^2*e*Sqrt[-c + (d*e)/f]*h)
/(c + d*x)^2 + (A*b^4*c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (a^2*b^2*c
^3*C*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (a*A*b^3*c^2*d*Sqrt[-c + (d*e)/
f]*f*h)/(c + d*x)^2 - (a^3*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 +
 (A*b^4*c*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (a^2*b^2*c*C*d*Sqrt[-c + (
d*e)/f]*f*g)/(c + d*x) - (a*A*b^3*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) -
(a^3*b*C*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (A*b^4*c*d*e*Sqrt[-c + (d
*e)/f]*h)/(c + d*x) + (a^2*b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (
a*A*b^3*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (a^3*b*C*d^2*e*Sqrt[-c ...
```

3.30.3 Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2108, 25, 2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 2108$$

$$\frac{\int -\frac{(2Adfh-C(deg+cfg+ceh))a^2+2b(cCeg-Adfg-Adeh-Acfh)a-\left(Ca^2+Ab^2\right)dfhx^2+Ab^2(deg+cfg+ceh)-2\left(C(dfg+deh+cfh)a^2+b(Adfh-\right.}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2(bc-ad)(be-af)(bg-ah)}$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{(2Adfh-C(deg+cfg+ceh))a^2+2b(cCeg-Adfg-Adeh-Acfh)a-(Ca^2+Ab^2)dfhx^2+Ab^2(deg+cfg+ceh)-2(C(dfg+deh+cfh)a^2+b(Adfh-}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2(bc-ad)(be-af)(bg-ah)}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 2110

$$-\frac{\int \frac{\frac{Cdfha^3}{b^2}-\frac{2Cdfga^2}{b}-\frac{2Cdeha^2}{b}-\frac{2cCfha^2}{b}+2Cdega+2cCfga+2cCeha-Adfha-2bcCeg+\left(-\frac{Cdfha^2}{b}-Abdfh\right)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{(a^4Cdfh-2a^3bC(cfh+}{}}{2(bc-ad)(be-a}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 176

$$-\frac{-\frac{(bg-ah)(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf))\int\frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2}-df\left(\frac{a^2C}{b}+Ab\right)\int\frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx-\frac{(a^4Cdfh-2a^3bC(}{}}{2(bc-ad)(be-}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 124

$$-\frac{-\frac{(bg-ah)(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf))\int\frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2}-\frac{df\sqrt{g+hx}\left(\frac{a^2C}{b}+Ab\right)\sqrt{\frac{d(e+fx)}{de-cf}}\int\frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-}{2(bc-}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 123

$$-\frac{-\frac{(bg-ah)(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf))\int\frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2}-\frac{(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+}{}}{2}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 131

3.30. $\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{-\frac{(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b^2\sqrt{e+fx}}-\frac{\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(\right.}{}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 131

$$-\frac{-\frac{(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b^2\sqrt{e+fx}\sqrt{g+hx}}-\frac{\left(a^4Cdfh-2a^3bC(cfh\right.}{}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 130

$$-\frac{-\frac{\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))-2ab^3(-Acfh-Adeh-Adfg+2cCeg)-Ab^4(ceh+cfg+deg)\right)\int \frac{}{(a+bx)\sqrt{c-}}}{b^2}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 187

$$-\frac{\frac{2\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))-2ab^3(-Acfh-Adeh-Adfg+2cCeg)-Ab^4(ceh+cfg+deg)\right)\int \frac{}{(bc-ad-b(c}}{b^2}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 413

$$-\frac{\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))-2ab^3(-Acfh-Adeh-Adfg+2cCeg)-Ab^4(ceh+cfg+deg)\right)}{b^2\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 413

3.30. $\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))-2ab^3(-Acfh-Adeh-Adfg+2cCeg)-Ab^4(ceh\right.}{b^2\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{412}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{(a+bx)(bc-ad)(be-af)(bg-ah)}-$$

$$\frac{-\frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\sqrt{f}\sqrt{g+hx}\left(\frac{a^2C}{b}\right.}{}}{}$$

input
```
Int[(A + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
-(((A*b^2 + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)
*(b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((-2*(A*b + (a^2*C)/b)*Sqrt[f]*Sqrt
[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[Arc
Sin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
 c*h))])/(Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e)
+ c*f]*(a^2*C*d*f - 2*a*b*C*(d*e + c*f) + b^2*(2*c*C*e - A*d*f))*(b*g - a*
h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*Ellipti
cF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*
(d*g - c*h))])/(b^2*d*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]) + (2*Sqrt[-(d*e
) + c*f]*(a^4*C*d*f*h - A*b^4*(d*e*g + c*f*g + c*e*h) - 2*a^3*b*C*(d*f*g +
 d*e*h + c*f*h) - 2*a*b^3*(2*c*C*e*g - A*d*f*g - A*d*e*h - A*c*f*h) - 3*a^
2*b^2*(A*d*f*h - C*(d*e*g + c*f*g + c*e*h)))*Sqrt[1 + (f*(c + d*x))/(d*e -
 c*f)]*Sqrt[1 + (h*(c + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((
b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e
- c*f)*h)/(f*(d*g - c*h))])/(b^2*(b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f
*(c + d*x))/d]*Sqrt[g - (c*h)/d + (h*(c + d*x))/d]))/(2*(b*c - a*d)*(b*e -
 a*f)*(b*g - a*h))
```

3.30. $\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2108
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*
(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(
d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(
m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*
g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^
2 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && I
ntegerQ[2*m] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

3.30. $\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.30.4 Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 1269, normalized size of antiderivative = 1.72

method	result	size
elliptic	Expression too large to display	1269
default	Expression too large to display	17416

input
```
int((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(1/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+
a*b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*
x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)+2*(C/b^2-1/2*a/b^2*d*f*h*
(A*b^2+C*a^2)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a
*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x
+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*
e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/
(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-d*f*h*(A*b^2+C*a^2)/(a^3*d
*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e
*g-b^3*c*e*g)/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(
1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c
*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/
f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))
^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2)))+(3*A*a^2*b^2*d*f*h-2*A*a*b^3*c*f*h-
2*A*a*b^3*d*e*h-2*A*a*b^3*d*f*g+A*b^4*c*e*h+A*b^4*c*f*g+A*b^4*d*e*g-C*a^4*
d*f*h+2*C*a^3*b*c*f*h+2*C*a^3*b*d*e*h+2*C*a^3*b*d*f*g-3*C*a^2*b^2*c*e*h-3*
C*a^2*b^2*c*f*g-3*C*a^2*b^2*d*e*g+4*C*a*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-
a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^3
*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f...
```

3.30.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="fricas")
```

output
```
Timed out
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

output
```
Timed out
```

3.30.7 Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

3.30.8 Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Cx^2}{(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}} \, dx = \int \frac{C\,x^2 + A}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,(a + b\,x)^2\,\sqrt{c + d\,x}} \, dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/
2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/
2)), x)
```

3.31 $\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.31.1 Optimal result

Integrand size = 44, antiderivative size = 1395

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{(C(3adfh-5b(dfg+deh+cfh))(adfh-3b(dfg+deh+cfh))+8bdf}{24}$$

$$+\frac{C(3adfh-5b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{12d^2f^2h^2}$$

$$+\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(C(3adfh-5b(dfg+deh+cfh))(adfh-3b(dfg+deh+cfh))+8bdfh(3Abdfh-}{24bd^3f^3h^3\sqrt{}}$$

$$+\frac{(be-af)\sqrt{bg-ah}(3a^2Cd^2f^2h^2+6abCdfh(cfh+2d(fg+eh))-b^2(24Ad^2f^2h^2+C(5c^2f^2h^2+4cdfh(f}{24b^2d^2f^3h^3\sqrt{fg}}$$

$$-\frac{\sqrt{-dg+ch}(4bdfh(C(b(deg+cfg+ceh)+a(dfg+deh+cfh))(3adfh-5b(dfg+deh+cfh))+2dfh(}{}$$

output
```
-1/24*(4*b*d*f*h*(C*(b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))*(3*a*d*f
*h-5*b*(c*f*h+d*e*h+d*f*g))+2*d*f*h*(3*b^2*c*C*e*g+2*a^2*C*(c*f*h+d*e*h+d*
f*g)-a*b*(12*A*d*f*h-5*C*(c*e*h+c*f*g+d*e*g))))+(a*d*f*h+b*(c*f*h+d*e*h+d*
f*g))*(C*(3*a*d*f*h-5*b*(c*f*h+d*e*h+d*f*g))*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f
*g))+8*b*d*f*h*(3*A*b*d*f*h-C*(2*b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*
g)))))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(
b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/
(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))
^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b^2/d^3/f^3/h^4/(-a*d
+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/24*(C*(3*a*d*f*h-5*b*(c*f*h+d*e*
h+d*f*g))*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))+8*b*d*f*h*(3*A*b*d*f*h-C*(2*b*
(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))))*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(
h*x+g)^(1/2)/b/d^2/f^3/h^3/(d*x+c)^(1/2)+1/3*C*(b*x+a)^(3/2)*(d*x+c)^(1/2)
*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+1/12*C*(3*a*d*f*h-5*b*(c*f*h+d*e*h+d*f*
g))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+1/
24*(-a*f+b*e)*(3*a^2*C*d^2*f^2*h^2+6*a*b*C*d*f*h*(c*f*h+2*d*(e*h+f*g))-b^2
*(24*A*d^2*f^2*h^2+C*(5*c^2*f^2*h^2+4*c*d*f*h*(e*h+f*g)+d^2*(15*e^2*h^2+14
*e*f*g*h+15*f^2*g^2))))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g
)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2)
)*(-a*h+b*g)^(1/2)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g...
```

3.31.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 39032 vs. $2(1395) = 2790$.

Time = 40.08 (sec) , antiderivative size = 39032, normalized size of antiderivative = 27.98

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]),x]
```

output
```
Result too large to show
```

3.31.3 Rubi [A] (warning: unable to verify)

Time = 5.28 (sec) , antiderivative size = 1389, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2104, 25, 2103, 2105, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \; 2104$$

$$\frac{\int -\frac{\sqrt{a+bx}(-C(3adfh-5b(dfg+deh+cfh))x^2-2(3Abdfh-2bC(deg+cfg+ceh)-aC(dfg+deh+cfh))x+3bcCeg-6aAdfh+aC(deg+cfg+ceh))}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6dfh} \cdots$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 25$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \frac{\int \frac{\sqrt{a+bx}(-C(3adfh-5b(dfg+deh+cfh))x^2-2(3Abdfh-2bC(deg+cfg+ceh)-aC(dfg+deh+cfh))x+3bcCeg-6aAdfh+aC(deg+cfg+ceh))}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{6dfh}$$

$$\Big\downarrow \; 2103$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \int \frac{-\left(\left(24Ad^2f^2h^2b^2+15C(dfg+deh+cfh)^2b^2-16Cdfh(deg+cfg+ceh)b^2-22aCdfh(dfg+deh+cfh)b+3a^2Cd^2f^2h^2\right)x^2\right)+2\left(C(b(deg+cfg+ceh)+a(dfg+deh+cfh \cdots\right.}{\cdots}$$

$$\Big\downarrow \; 2105$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \frac{-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}} - \frac{(de-cf)(dg-ch)\left(24Ad^2f^2h^2b^2+15 \cdots\right.}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 194$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\int\frac{(bdeg+acfh)\left(24Ad^2f^2h^2b^2+15C\right.}{}$$

↓ 327

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+15\right.}{}$$

↓ 2101

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+15\right.}{}$$

↓ 183

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+15\right.}{}$$

↓ 188

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+15\right.}{}$$

↓ 321

3.31. $\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+15\right.}{}$$

$$\big\downarrow \text{412}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+15\right.}{}$$

input
```
Int[((a + b*x)^(3/2)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
(C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) -
(-1/2*(C*(3*a*d*f*h - 5*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h) + (-(((24*A*b*d^2*f*h + (3*a^2*C
*d^2*f*h)/b - 16*b*C*d*(d*e*g + c*f*g + c*e*h) - 22*a*C*d*(d*f*g + d*e*h +
 c*f*h) + (15*b*C*(d*f*g + d*e*h + c*f*h)^2)/(f*h))*Sqrt[a + b*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x])/Sqrt[c + d*x]) + (Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(24
*A*b^2*d^2*f^2*h^2 + 3*a^2*C*d^2*f^2*h^2 - 16*b^2*C*d*f*h*(d*e*g + c*f*g +
 c*e*h) - 22*a*b*C*d*f*h*(d*f*g + d*e*h + c*f*h) + 15*b^2*C*(d*f*g + d*e*h
 + c*f*h)^2)*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c
+ d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h
]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(
b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x
]) + ((-2*d*(b*e - a*f)*Sqrt[b*g - a*h]*(3*a^2*C*d^2*f^2*h^2 + 6*a*b*C*d*f
*h*(c*f*h + 2*d*(f*g + e*h)) - b^2*(24*A*d^2*f^2*h^2 + C*(5*c^2*f^2*h^2 +
4*c*d*f*h*(f*g + e*h) + d^2*(15*f^2*g^2 + 14*e*f*g*h + 15*e^2*h^2))))*Sqrt
[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[
ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -
(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*Sqrt[f*g - e*h]
*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) +
 (2*Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(24*A*b^2...
```

3.31. $\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.31. $\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2104
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f
*g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
 c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
 1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

3.31. $\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.31.4 Maple [A] (verified)

Time = 6.74 (sec) , antiderivative size = 2228, normalized size of antiderivative = 1.60

method	result	size
elliptic	Expression too large to display	2228
default	Expression too large to display	92114

input
```
int((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(1/3*C*b/d/f/h*x*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^
3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+
b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(
1/2)+1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*e*h+5/2*b
*d*f*g))/b/d/f/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*
x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^
2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2*(a^2*A-1/3*C*b/
d/f/h*a*c*e*g-1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*
e*h+5/2*b*d*f*g))/b/d/f/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g
))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/
d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x
+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h
))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d
)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(2*a*b*A-1/3*C*b/d/f/h*(3/2*a*
c*e*h+3/2*a*c*f*g+3/2*a*d*e*g+3/2*b*c*e*g)-1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2
*a*d*f*h+5/2*b*c*f*h+5/2*b*d*e*h+5/2*b*d*f*g))/b/d/f/h*(a*c*f*h+a*d*e*h+a*
d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(
x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c
/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*...
```

3.31.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
Timed out
```

3.31.6 Sympy [F]

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Integral((A + C*x**2)*(a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
 + h*x)), x)
```

3.31.7 Maxima [F]

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

3.31.8 Giac [F]

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(C\,x^2+A)\,(a+b\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + C*x^2)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*
x)^(1/2)),x)
```

output
```
int(((A + C*x^2)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*
x)^(1/2)), x)
```

3.32 $\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.32.1 Optimal result

Integrand size = 44, antiderivative size = 937

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{C(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{4bdf^2h^2\sqrt{c+dx}}$$

$$+ \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$- \frac{C\sqrt{dg-ch}\sqrt{fg-eh}(adfh-3b(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|}{4bd^2f^2h^2\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$+ \frac{C(be-af)\sqrt{bg-ah}(adfh+b(cfh+3d(fg+eh)))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{}}{\sqrt{fg-eh}\sqrt{}}\right.\right.}{4b^2df^2h^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$- \frac{\sqrt{-dg+ch}(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh-C(b(deg}{4}$$

output
```
-1/4*(C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))-
4*b*d*f*h*(2*A*b*d*f*h-C*(b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))))*(
b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(
1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*
g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*(
(-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b^2/d^2/f^2/h^3/(-a*d+b*c)^(1
/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+1/4*C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(b
*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b/d/f^2/h^2/(d*x+c)^(1/2)+1/2*C*(b
*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+1/4*C*(-a*f+b*
e)*(a*d*f*h+b*(c*f*h+3*d*(e*h+f*g)))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1
/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*
h+b*g))^(1/2))*(-a*h+b*g)^(1/2)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1
/2)*(h*x+g)^(1/2)/b^2/d/f^2/h^2/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e
)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-1/4*C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g)
)*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),
((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*
h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/
b/d^2/f^2/h^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

3.32.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 16972 vs. $2(937) = 1874$.

Time = 36.21 (sec) , antiderivative size = 16972, normalized size of antiderivative = 18.11

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
Result too large to show
```

3.32.3 Rubi [A] (warning: unable to verify)

Time = 2.69 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2104, 2105, 25, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \ 2104$$

$$\frac{\int \frac{C(adfh-3b(dfg+deh+cfh))x^2+2(2Abdfh-C(b(deg+cfg+ceh)+a(dfg+deh+cfh)))x+4aAdfh-C(bceg+a(deg+cfg+ceh))}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{4dfh}+\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 2105$$

$$\frac{\frac{\int -\frac{C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))-2bdfh(4aAdfh-C(bceg+a(deg+cfg+ceh)))+(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}}{}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))+(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}}{}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 194$$

$$\frac{-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))+(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}}{}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 327$$

$$-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))+(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 2101

$$-\frac{\frac{(C(adfh-3b(cfh+deh+dfg))(adfh+b(cfh+deh+dfg))-4bdfh(2Abdfh-C(a(cfh+deh+dfg)+b(ceh+cfg+deg))))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{Cd(be-af)(bg-\cdots}{}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 183

$$-\frac{\frac{2(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(C(adfh-3b(cfh+deh+dfg))(adfh+b(cfh+deh+dfg))-4bdfh(2Abdfh-C(a(cfh+deh+dfg)+b(ceh+cfg+deg))))\int\cdots}{b\sqrt{c+dx}\sqrt{e+fx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 188

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(adfh-3b(dfg+deh+cfh))}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+\cdots}}{bf\cdots}$$

$\downarrow$ 321

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(adfh-3b(dfg+deh+cfh))}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+\cdots}}{bf\cdots}$$

$\downarrow$ 412

3.32. $\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(adfh-3b(dfg+deh+cfh))}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+h}}{bf}$$

input
```
Int[(Sqrt[a + b*x]*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h) + ((
C*(a*d*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt
[g + h*x])/(b*f*h*Sqrt[c + d*x]) - (C*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(a*d
*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g +
 h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e
+ f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e
- a*f)*(d*g - c*h))])/(b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(
c + d*x))]*Sqrt[g + h*x]) - ((-2*C*d*(b*e - a*f)*Sqrt[b*g - a*h]*(b*c*f*h
+ a*d*f*h + 3*b*d*(f*g + e*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(
a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/
(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)
*(b*g - a*h)))])/(b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g +
 h*x))/((f*g - e*h)*(a + b*x)))]) + (2*Sqrt[-(d*g) + c*h]*(C*(a*d*f*h - 3*
b*(d*f*g + d*e*h + c*f*h))*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)) - 4*b*d*f
*h*(2*A*b*d*f*h - C*(b*(d*e*g + c*f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h)
)))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[(
(b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h
))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) +
 c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))
])/(b*Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x]))/(2*b*d*f*h))/(4*d...
```

3.32. $\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2104
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f
*g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
 c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
 1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

3.32. $\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} dx$

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. $2(854) = 1708$.

Time = 5.23 (sec) , antiderivative size = 1794, normalized size of antiderivative = 1.91

method	result	size
elliptic	Expression too large to display	1794
default	Expression too large to display	43214

input
```
int((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(1/2*C/d/f/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*
d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*
f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)
+2*(A*a-1/2*C/d/f/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(g/
h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)
*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))
^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/
2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h
-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(A*b-1/2*C/d/f/h*(a*c*f*h+a*d*e*h+a*
d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(
x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c
/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(
x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/
(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2
))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/
h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+(C*a
-1/2*C/d/f/h*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*b*d*f*g))*((x+a/b)*(
x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+
c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h...
```

3.32.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
Timed out
```

3.32.6 Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Integral((A + C*x**2)*sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g +
h*x)), x)
```

3.32.7 Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.32.8 Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(C\,x^2+A)\,\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + C*x^2)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*
x)^(1/2)),x)
```

output
```
int(((A + C*x^2)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*
x)^(1/2)), x)
```

3.33 $\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.33.1 Optimal result

Integrand size = 44, antiderivative size = 757

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$
$$-\frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$+\frac{(a^2Cfh+abC(fg+eh)-b^2(Ceg-2Afh))\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),\right.}{b^2fh\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$-\frac{C\sqrt{-dg+ch}(adfh+b(dfg+deh+cfh))(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\right.}{b^2d\sqrt{bc-ad}fh^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-C*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*
x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f
+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)
*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))
^(1/2)/b^2/d/f/h^2/(-a*d+b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)+C*(b*x+a)^
(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b/f/h/(d*x+c)^(1/2)+(a^2*C*f*h+a*b*C*(e*
h+f*g)-b^2*(-2*A*f*h+C*e*g))*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*
h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^
(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/b^2/f/h
/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*
h+f*g)/(b*x+a))^(1/2)-C*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g
)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))
*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*
h+f*g)/(d*x+c))^(1/2)/b/d/f/h/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2
)/(h*x+g)^(1/2)
```

3.33.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6321 vs. $2(757) = 1514$.

Time = 34.84 (sec) , antiderivative size = 6321, normalized size of antiderivative = 8.35

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
Result too large to show
```

3.33.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2106, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 2106$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)-C(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{C(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 194$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)-C(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-\frac{C\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 327$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)-C(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 2101$$

$$\frac{\frac{d\left(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{C(adfh+b(cfh+deh+dfg))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}}{2bdfh}-\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 183$$

3.33. $\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$

$$\frac{\frac{d\left(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(adfh+b(cfh+deh+dfg}{b\sqrt{c+dx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{188}$$

$$\frac{\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\left(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh)\right)\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-}{(a+bx)(dg-}}}{}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\left(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2C(a+bx)\sqrt{\frac{(c+dx)(}{(a+bx)(}}}{}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{412}$$

$$\frac{\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\left(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2C(a+bx)\sqrt{ch-dg}}{}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

3.33. $\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

input
```
Int[(A + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*x]) - (C*S
qrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x)
)/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x
])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f
)*(d*g - c*h))])/(b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d
*x))]*Sqrt[g + h*x]) + ((2*d*(a^2*C*f*h + a*b*C*(f*g + e*h) - b^2*(C*e*g -
 2*A*f*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g +
h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqr
t[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*
Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x
))/((f*g - e*h)*(a + b*x)))]) - (2*C*Sqrt[-(d*g) + c*h]*(a*d*f*h + b*(d*f*
g + d*e*h + c*f*h))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
 + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi
[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x]
)/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a
*d)*(f*g - e*h))])/(b*Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x]))/(2*b
*d*f*h)
```

3.33.3.1 Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2106
```
Int[((A_.) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[C*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x])), x] + (Si
mp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) - C*(a*d*f*h + b*(d*f*g
+ d*e*h + c*f*h))*x, x], x], x] + Simp[C*(d*e - c*f)*((d*g - c*h)/(2*b*d*f*
h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x],
x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x]
```

3.33. $\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.33.4 Maple [A] (verified)

Time = 6.37 (sec) , antiderivative size = 1065, normalized size of antiderivative = 1.41

method	result	size
elliptic	Expression too large to display	1065
default	Expression too large to display	15875

input
```
int((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*A*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d
))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/
b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b
)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/
(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+C*((x+a/
b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)
*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/
h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)
/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-
c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c
/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/
d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+
c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-
c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+
a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

3.33.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
Timed out
```

3.33.6 Sympy [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Integral((A + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g +
h*x)), x)
```

3.33.7 Maxima [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.33.8 Giac [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Cx^2}{\sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{C\,x^2 + A}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,\sqrt{a + b\,x}\,\sqrt{c + d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)
^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)
^(1/2)), x)
```

3.34 $\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.34.1 Optimal result

Integrand size = 44, antiderivative size = 867

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(Ab^2+a^2C)\,d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{b(bc-ad)(be-af)(bg-ah)\sqrt{c+dx}}$$
$$-\frac{2(Ab^2+a^2C)\,\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$
$$-\frac{2(Ab^2+a^2C)\,\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\,E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)\middle|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{b(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$
$$-\frac{2(2abcC+Ab^2d-a^2Cd)\,\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b^2(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$+\frac{2C\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\mathrm{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-af)(\cdots}{(bc-ad)(f\cdots}\right.}{b^2\sqrt{bc-ad}\,h\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2*C*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x
+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e
*h+f*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1
/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/b^2/h/(-a*d+b*c)^(1/2)/(
d*x+c)^(1/2)/(f*x+e)^(1/2)+2*(A*b^2+C*a^2)*d*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(
h*x+g)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(d*x+c)^(1/2)-2*(A*b^2+C*a
^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+
b*g)/(b*x+a)^(1/2)-2*(A*b^2*d-C*a^2*d+2*C*a*b*c)*EllipticF((-a*h+b*g)^(1/2
)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c
*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(
h*x+g)^(1/2)/b^2/(-a*d+b*c)/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2
)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-2*(A*b^2+C*a^2)*EllipticE
((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2),((-a*d+b*c)
*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2
)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1/2)/b/(-a*d+b*c
)/(-a*f+b*e)/(-a*h+b*g)/((-c*f+d*e)*(b*x+a)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x
+g)^(1/2)
```

3.34.2 Mathematica [A] (warning: unable to verify)

Time = 31.92 (sec) , antiderivative size = 721, normalized size of antiderivative = 0.83

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left(2aC(-bc+ad)h(-bg+ah)\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+a}{(fg-eh}}\right.\right.\right.}{}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
(-2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
 f*x)^(3/2)*(g + h*x)^(3/2)*(2*a*C*(-(b*c) + a*d)*h*(-(b*g) + a*h)*Ellipti
cF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b
*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] - A*b^2*h*(b*(d*g -
c*h)*EllipticE[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*
x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] + d*(-(b
*g) + a*h)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(
a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]) -
 a^2*C*h*(b*(d*g - c*h)*EllipticE[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/(
(f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*
g - c*h))] + d*(-(b*g) + a*h)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h
*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*
f)*(d*g - c*h))]) + C*(b*c - a*d)*(b*g - a*h)^2*EllipticPi[(b*(-(f*g) + e*
h))/((b*e - a*f)*h), ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(
a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]))/
(b^2*(b*c - a*d)*h*(f*g - e*h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e -
a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

3.34.3 Rubi [A] (warning: unable to verify)

Time = 2.35 (sec) , antiderivative size = 859, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2108, 25, 2105, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2108}$$

$$\frac{\int -\frac{-2(Ca^2+Ab^2)dfhx^2-(2C(dfg+deh+cfh)a^2+b(Adfh-C(deg+cfg+ceh))a+b^2(cCeg+Adfg+Adeh+Acfh))x+a(aAdfh-aC(deg+cfg+ce}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{-2(Ca^2+Ab^2)dfhx^2-(2C(dfg+deh+cfh)a^2+b(Adfh-C(deg+cfg+ceh))a+b^2(cCeg+Adfg+Adeh+Acfh))x+a(aAdfh-aC(deg+cfg+ce}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 2105

$$-\frac{-\frac{(a^2C+Ab^2)(de-cf)(dg-ch)\int\frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{\int\frac{2dfh((acC+Abd)(be-af)(bg-ah)-C(bc-ad)(be-af)(bg-ah)x)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-\frac{2d\sqrt{a+bx}\sqrt{e}}{}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 27

$$-\frac{-\frac{(a^2C+Ab^2)(de-cf)(dg-ch)\int\frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{\int\frac{(acC+Abd)(be-af)(bg-ah)-C(bc-ad)(be-af)(bg-ah)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{}}{b\sqrt{c}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 194

$$-\frac{\frac{2\sqrt{a+bx}(a^2C+Ab^2)(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int\frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{b\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{\int\frac{(acC+Abd)(be-af)(bg-ah)-C(bc-ad)(be-af)(bg-ah)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 327

$$-\frac{\frac{\int\frac{(acC+Abd)(be-af)(bg-ah)-C(bc-ad)(be-af)(bg-ah)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2\sqrt{a+bx}(a^2C+Ab^2)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right.\right.}{b\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 2101

3.34. $\int\frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{\frac{\frac{(be-af)(bg-ah)\left(a^2(-C)d+2abcC+Ab^2d\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{C(bc-ad)(be-af)(bg-ah)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}}{b}+\frac{2\sqrt{a+bx}\left(a^2C+Ab\right.}{\ }}{(bc-ad)(be-af)(bg}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 183$$

$$-\frac{\frac{\frac{(be-af)(bg-ah)\left(a^2(-C)d+2abcC+Ab^2d\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{2C(a+bx)(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{\ }{\left(h-\right.}}{b\sqrt{c+dx}\sqrt{e+fx}}}{b}}{\ }$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 188$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$$\frac{-\frac{2\left(Ca^2+Ab^2\right)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}+\frac{2\left(Ca^2+Ab^2\right)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{\ }$$

$$\Big\downarrow \; 321$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$$\frac{-\frac{2\left(Ca^2+Ab^2\right)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}+\frac{2\left(Ca^2+Ab^2\right)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{\ }$$

$$\Big\downarrow \; 412$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$$\frac{-\frac{2\left(Ca^2+Ab^2\right)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}+\frac{2\left(Ca^2+Ab^2\right)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{\ }$$

3.34. $\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

input
```
Int[(A + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(-2*(A*b^2 + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) - ((-2*(A*b^2 + a^2*C)*d*Sqrt[a +
 b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*Sqrt[c + d*x]) + (2*(A*b^2 + a^2*C)*
Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x
))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*
x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*
f)*(d*g - c*h))])/(b*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]
*Sqrt[g + h*x]) + ((2*(2*a*b*c*C + A*b^2*d - a^2*C*d)*(b*e - a*f)*Sqrt[b*g
 - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x
]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a
 + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*Sqr
t[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a
+ b*x)))]) - (2*C*Sqrt[b*c - a*d]*(b*e - a*f)*(b*g - a*h)*Sqrt[-(d*g) + c*
h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((
b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h)
)/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) +
c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))]
)/(b*h*Sqrt[c + d*x]*Sqrt[e + f*x]))/b)/((b*c - a*d)*(b*e - a*f)*(b*g - a*
h))
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

3.34. $\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2108
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*
(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(
d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(
m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*
g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^
2 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && I
ntegerQ[2*m] && LtQ[m, -1]
```

3.34.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(794) = 1588$.

Time = 7.83 (sec) , antiderivative size = 2286, normalized size of antiderivative = 2.64

method	result	size
elliptic	Expression too large to display	2286
default	Expression too large to display	33894

input
```
int((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+
b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*
h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2)
/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f
*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(-C*a/b^2+1/b^2*(a^2*d*f*h-a*b*c*f*h-a*b*
d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(A*b^2+C*a^2)/(a^3*d*f*h-a^
2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*
c*e*g)-(b*c*e*h+b*c*f*g+b*d*e*g)/b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*
b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2))*(g/h
-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*
(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^
(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2
)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-
a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(C/b-1/b*(a*d*f*h-b*c*f*h-b*d*e*h-b*d
*f*g)*(A*b^2+C*a^2)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c
*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/b/
(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b
^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b
)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2...
```

3.34.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
Timed out
```

3.34.6 Sympy [F]

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Integral((A + C*x**2)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
 + h*x)), x)
```

3.34.7 Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

3.34.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)
^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)
^(1/2)), x)
```

3.35 $\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

3.35.1 Optimal result

Integrand size = 44, antiderivative size = 1070

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{4d(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(2Ad(fg}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{c+dx}}$$

$$-\frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$+\frac{4b(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2(2Ad(fg}{3(bc-ad)^2(be-af)^2(bg-ah)^2\sqrt{a+bx}}$$

$$+\frac{4\sqrt{dg-ch}\sqrt{fg-eh}(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ce}{3(bc-ad)^2(be-af)^2(bg}$$

$$-\frac{2(3ab(c^2C+Ad^2)(fg+eh)-b^2(2Ad^2eg+Acd(fg+eh)+c^2(3Ceg-Afh))-a^2(3Ad^2fh-C(d^2eg-}{3(bc-ad)^2(be-af)(bg-ah)^{3/2}\sqrt{fg}}$$

output
```
-4/3*d*(A*b^3*(c*e*h+c*f*g+d*e*g)+a^3*C*(c*f*h+d*e*h+d*f*g)+a^2*b*(3*A*d*f
*h-2*C*(c*e*h+c*f*g+d*e*g))-a*b^2*(2*A*d*(e*h+f*g)-c*(-2*A*f*h+3*C*e*g)))*
(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+
b*g)^2/(d*x+c)^(1/2)-2/3*(A*b^2+C*a^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)
^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)+4/3*b*(A*b^3*(c*e*h+
c*f*g+d*e*g)+a^3*C*(c*f*h+d*e*h+d*f*g)+a^2*b*(3*A*d*f*h-2*C*(c*e*h+c*f*g+d
*e*g))-a*b^2*(2*A*d*(e*h+f*g)-c*(-2*A*f*h+3*C*e*g)))*(d*x+c)^(1/2)*(f*x+e)
^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/(b*x+a)^(1/2)-
2/3*(3*a*b*(A*d^2+C*c^2)*(e*h+f*g)-b^2*(2*A*d^2*e*g+A*c*d*(e*h+f*g)+c^2*(-
A*f*h+3*C*e*g))-a^2*(3*A*d^2*f*h-C*(-2*c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)))*
EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-
(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-
c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)^(
3/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a
))^(1/2)+4/3*(A*b^3*(c*e*h+c*f*g+d*e*g)+a^3*C*(c*f*h+d*e*h+d*f*g)+a^2*b*(3
*A*d*f*h-2*C*(c*e*h+c*f*g+d*e*g))-a*b^2*(2*A*d*(e*h+f*g)-c*(-2*A*f*h+3*C*e
*g)))*EllipticE((-c*h+d*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1
/2),((-a*d+b*c)*(-e*h+f*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-c*h+d*g)^(1/2)*
(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*(-(-c*f+d*e)*(h*x+g)/(-e*h+f*g)/(d*x+c))^(1
/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/((-c*f+d*e)*(b*x+a)/(-a*f+b*...
```

3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11363 vs. $2(1070) = 2140$.

Time = 40.54 (sec) , antiderivative size = 11363, normalized size of antiderivative = 10.62

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
Result too large to show
```

3.35.3 Rubi [A] (warning: unable to verify)

Time = 3.80 (sec) , antiderivative size = 1057, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2108, 25, 2102, 25, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2108}$$

$$\frac{\int -\frac{(3Adfh-C(deg+cfg+ceh))a^2+3b(cCeg-Adfg-Adeh-Acfh)a+2Ab^2(deg+cfg+ceh)-(2C(dfg+deh+cfh)a^2+3b(Adfh-C(deg+cfg+ceh}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{(3Adfh-C(deg+cfg+ceh))a^2+3b(cCeg-Adfg-Adeh-Acfh)a+2Ab^2(deg+cfg+ceh)-(2C(dfg+deh+cfh)a^2+3b(Adfh-C(deg+cfg+ceh}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2102}$$

$$-\frac{\int -\frac{-4bdfh(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh))x^2-2(adfh+b(dfg+deh+cfh))(C}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int \frac{-4bdfh(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh))x^2-2(adfh+b(dfg+deh+cfh))(C}{}}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(a^2C+Ab^2)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2105}$$

$$-\frac{-\frac{\int -\frac{2bdf(be-af)h(bg-ah)\left(-\left(\left(3Ad^2fh-C\left(-2fhc^2-d(fg+eh)c+d^2eg\right)\right)a^2\right)+3b\left(Cc^2+Ad^2\right)(fg+eh)a-b^2\left((3Ceg-Afh)c^2+Ad(fg+eh)c+2Ad^2eg\right)\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-2}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 27

$$-\frac{-\frac{-(be-af)(bg-ah)\left(-\left(a^2\left(3Ad^2fh-C\left(-2c^2fh-cd(eh+fg)+d^2eg\right)\right)\right)+3ab\left(Ad^2+c^2C\right)(eh+fg)-b^2\left(c^2(3Ceg-Afh)+Acd(eh+fg)+2Ad^2eg\right)\right.}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 188

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$-\frac{4b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh)\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$\downarrow$ 194

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$-\frac{4b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh)\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$\downarrow$ 321

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$-\frac{4b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh)\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$\downarrow$ 327

3.35. $\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$-\frac{\frac{4b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh)\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} -}{}$$

input
```
Int[(A + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(-2*(A*b^2 + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a
*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) - ((-4*b*(A*b^3*(d*e*g + c*f*
g + c*e*h) + a^3*C*(d*f*g + d*e*h + c*f*h) + a^2*b*(3*A*d*f*h - 2*C*(d*e*g
 + c*f*g + c*e*h)) - a*b^2*(2*A*d*(f*g + e*h) - c*(3*C*e*g - 2*A*f*h)))*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a
*h)*Sqrt[a + b*x]) - ((-4*d*(A*b^3*(d*e*g + c*f*g + c*e*h) + a^3*C*(d*f*g
+ d*e*h + c*f*h) + a^2*b*(3*A*d*f*h - 2*C*(d*e*g + c*f*g + c*e*h)) - a*b^2
*(2*A*d*(f*g + e*h) - c*(3*C*e*g - 2*A*f*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*
Sqrt[g + h*x])/Sqrt[c + d*x] + (4*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(A*b^3*(
d*e*g + c*f*g + c*e*h) + a^3*C*(d*f*g + d*e*h + c*f*h) + a^2*b*(3*A*d*f*h
- 2*C*(d*e*g + c*f*g + c*e*h)) - a*b^2*(2*A*d*(f*g + e*h) - c*(3*C*e*g - 2
*A*f*h)))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d
*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*S
qrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqr
t[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - (2*(b*
e - a*f)*Sqrt[b*g - a*h]*(3*a*b*(c^2*C + A*d^2)*(f*g + e*h) - b^2*(2*A*d^2
*e*g + A*c*d*(f*g + e*h) + c^2*(3*C*e*g - A*f*h)) - a^2*(3*A*d^2*f*h - C*(
d^2*e*g - 2*c^2*f*h - c*d*(f*g + e*h))))*Sqrt[((b*e - a*f)*(c + d*x))/((d*
e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[
e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))...
```

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

3.35. $\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2108
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*
(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(
d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(
m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*
g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^
2 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && I
ntegerQ[2*m] && LtQ[m, -1]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3341 vs. $2(998) = 1996$.

Time = 10.35 (sec) , antiderivative size = 3342, normalized size of antiderivative = 3.12

method	result	size
elliptic	Expression too large to display	3342
default	Expression too large to display	106972

input
```
int((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e
)^(1/2)/(h*x+g)^(1/2)*(2/3/b^2/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*
f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2)*(b*d*f*h*
x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^
2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*
e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+4/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*
e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*
b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*
e*g)^2*(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f*g+A*
b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*a^3*d*e*h+C*a^3*d*f*g-2*C*
a^2*b*c*e*h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a*b^2*c*e*g)/((x+a/b)*(b*d
*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x
+b*c*e*g))^(1/2)+2*(C/b^2-1/3/b^2*(3*A*a*b^2*d*f*h-A*b^3*c*f*h-A*b^3*d*e*h
-A*b^3*d*f*g+3*C*a^3*d*f*h-C*a^2*b*c*f*h-C*a^2*b*d*e*h-C*a^2*b*d*f*g)/(a^3
*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d
*e*g-b^3*c*e*g)+2/3/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b
^2*c*f*g+b^2*d*e*g)*(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a
*b^2*d*f*g+A*b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*a^3*d*e*h+C*a
^3*d*f*g-2*C*a^2*b*c*e*h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a*b^2*c*e*g)/
(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+...
```

3.35.5 Fricas [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
integral((C*x^2 + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3*a*b^2*d
)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*b^2*d)*
e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a*b^2*c
 + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*h)*x^
3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f + (3*a
^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3*d)*e)
*g)*x), x)
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Timed out
```

3.35.7 Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

3.35.8 Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)
^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)
^(1/2)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```